THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Engineering and Technology in Conjunction with Kenya Institute of Highways and Building & Technology (KIHBT)

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

HIGHER DIPLOMA IN ELECTRICAL & ELECTRONIC ENGINEERING

EEA 3101: ENGINEERING MATHEMATICS I

END OF SEMESTER EXAMINATION

SERIES: APRIL 2012

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Answer booklet*
- *Mathematical table/Scientific Calculator*
- *Drawing Instruments*
- *A bridged Laplace Transforms Table*

This paper consists of **FIVE** questions. Answer any **THREE** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

Question One

$$y = \frac{1}{\sqrt{x}}$$

a) (i) Differentiate from first principle (3 marks)

$$y = 1n\left[x + \sqrt{x^2 + a^2}\right] \; is \; \frac{1}{\sqrt{x^2 + a^2}}$$

(ii) Show that the derivative of (4 marks)

b) Differentiate the following functions with respect to x

$y = 6^x$

(i) (3 marks)

$x^3 + y^3 - 3axy = 0$

(ii) (3 marks)

$ax^2 + by^2 + 2xy = 2$ $\quad \frac{d^2y}{dx^2} = \frac{2 - (ax + by)^2}{(x + by)^3}$

c) If , show that (7 marks)

Question Two

$f(t) = e^{2t} \; \cos 3t$

a) (i) Derive the Laplace Transforms of the function from first principles.

(ii) Determine the inverse Laplace Transforms of the following

$$\frac{2s^2 - 6s - 1}{(s-3)(s^2 - 2s + 5)}$$

(I)

$$\frac{s^2 - 6s - 64}{(s-2)(s^2 - 16)}$$

(II) (13 marks)

$$\frac{d^2q}{dt^2} + 9q = 102e^{-5t} - \sin 3t$$

b) Charge in a certain circuit is described by a different equation of the type .

$q = 3e^{-5t} + \frac{t}{6}\cos 3t - \frac{1}{6}\sin 3t$ $\quad q_o = 3$

Use Laplace Transforms to show that , given that Columbs,

$q_1 = \frac{-46}{3}$

per second (6 marks)

Question Three

a) Evaluate the following integrals

(i) $\int \frac{x^3}{(x-1)(x-2)(x-3)}dx$

(ii) $\int x1n(1+x)dx$ (11 marks)

b) (i) Use the substitution $x = a\tan\theta$ to show that $\int \frac{dx}{\left(x^2+a^2\right)^{\frac{3}{2}}} = \frac{1}{a^2}\sin\left[\tan^{-1}\left(\frac{x}{a}\right)\right]+c$

(iii) If the work done W in moving a certain body from $x=a$ $x=2a$ is given by $\int_a^{2a}\frac{kdx}{\left(x^2+a^2\right)^{\frac{3}{2}}}$,

show that $W=\frac{k}{a^2}\left[\frac{2}{\sqrt{5}}-\frac{1}{\sqrt{2}}\right]$ (9 marks)

Question Four

a) (i) Solve completely the following differential equation

$$(1+x)\frac{dy}{dx}+(1+2x)y=(1+x)^2$$

(ii) State the necessary and sufficient condition for an equation $Mdx+Ndy=0$ to be exact hence solve the following equation

$$\left(\frac{2x}{y}+5y^2-4x\right)dx+\left(3y^2-\frac{x^2}{y^2}+10xy\right)dy=0$$

(10 marks)

b) An e.m.f of E_o = 40 volts, a resistor of R = 30 Ohms, inductor of L = 50mH and a capacitor of 250mF are connected in series. Initially the circuit is dead:

(I) Form a differential equation involving
  (i) The current i and time t
  (ii) The charge q and time t
  (iii) Solve for q and I given that i(o) = q(o)=0

(II) Derive the expressions for the voltages across
  (i) Inductor L
  (ii) Capacitor C
  (iii) Resistance R (10 marks)

Question Five

a) The instantaneous current i passing through a circuit of resistance R and inductance L satisfies the differential equation $L\frac{di}{dt} + Ri = V_O \cos \omega t$. Where t is time and V_o and ω are constant. Show that $i = \frac{V_O}{\omega^2 L^2 + R^2}\{L\omega \sin \omega t + R\cos \omega t\} + Ce^{-\frac{R}{L}t}$

(10 marks)

b) Solve the differential equation $(D^2 + 4)y = 16\cos 2t + 12\cos 4t$ using the D-operator method given that $t = 0,\ y = o$ and $y = \frac{dy}{dt} = 4$.

(10 marks)