TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:
BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE
(BMCS – Y1 S1 BSSC –Y1 S1)

SMA 2100: DISCRETE MATHEMATICS

END OF SEMESTER EXAMINATION
SERIES: APRIL 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) (i) Define the term power set. **(1 mark)**

$A = \{1, 3\{1,3\}\}$

(ii) Let find the power set of A. **(2 marks)**

$A = \{1,2,3\}$ $B = \{1,2,3,4\}$ $f : A \rightarrow B$ $f(x) = x + 1$

b) Let and Let be defined by . Determine if the function f is surjective or injective. **(3 marks)**

$$1.26 + 2.3 + 3.4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

c) Proof by mathematics induction that **(5 marks)**

d) Negate each of the following statements:
    (i) There exist an integer that is not divisible by 5.   **(1 mark)**
    (ii) John is at the Library or he is not at the book store.   **(1 mark)**

e) Proof that a statement is logically equivalent to its contra positive.   **(4 marks)**

$$q \rightarrow (r \vee (-s))$$

f) Draw the truth table for   **(4 marks)**

g) Determine the validity of the following argument.   **(4 marks)**

$$-p \rightarrow q$$
$$w \wedge p$$
$$\frac{q}{p \rightarrow q}$$

h) Show by shading the region described by the following:
$$(A \backslash B)^c \cap (B \cup C)^c$$

    (3 marks)

i) Prove that the square of an even number is even.   **(2 marks)**

Question Two

a) Define the term "mapping"   **(2 marks)**

$$f : \Re \rightarrow \Re \qquad f(a) = a+1 \qquad gb = \begin{cases} b & b < 5 \\ b-1 & b \geq 5 \end{cases}$$

b) Let f be a function   , defined by   by   compute:
    f^{-1}

    (i)   **(1 mark)**
      $f^{-1}(-2)$

    (ii)   **(1 mark)**
      gof

    (iii)   **(2 marks)**
      $g_o f(3)$

    (iv)   **(2 marks)**
      $g_o f_o f_o g_o f(2)$

    (v)   **(2 marks)**

c) Determine the validity of the following argument. If Akinyi bought a house then either she sold her car or she borrowed money from Muiganania Sacco. Akinyi did not sell her or she did not by a house. Akinyi has not borrowed money from Muiganania Sacco. Therefore if Akinyi did not sell her car then she did not buy a house.   **(10 marks)**

Question Three

a) (i) Define the term "Cartesian Product" of two sets A and B   **(2 marks)**

$$A = \{3,4\} \qquad B = \{3,6,4\}$$

(ii) Define A x B and B x A given that   and   **(3 marks)**

b) State any THREE De-Morgan;s laws of sets.   **(3 marks)**

c) Out of 300 students taking discrete Mathematics, 60 take coffee, 27 Coca, 36 take tea, 17 take tea only, 47 take chocolate tea and Cocoa, 20 take Cocoa only 2 take tea, coffee and chocolate 30 take coffee only 9 take tea and chocolate whereas 12 take tea and coffee:
   (i) Express this information on a Venn diagram.   **(6 marks)**
   (ii) Find how many take cocoa and tea only   **(2 marks)**
   (iii)   Find how many take any beverage   **(2 marks)**
   (iv) Find how many take at least two beverages   **(2 marks)**

Question Four

a) Use Mathematical induction to proof that:

$$\sum_{k-1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$$

                                                                                    (8 marks)

b) Proof that the:
   (i) Product of any two rational number is rational.   **(4 marks)**
   (ii) Product of any TWO irrational numbers is not always irrational.   **(2 marks)**

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

c) Prove that   **(4 marks)**

d) List all positive prime numbers less than 30   **(2 marks)**

Question Five

a) Determine whether the statement is a tantology a contingency or contradiction:
$$p \to (q \to p)$$

                                                                                    (5 marks)

b) Show that the set 24 of all even integers is countable.   **(5 marks)**

c) What is the contra positive, the inverse and the converse of:
    "If you have flue then you will miss the final examination"   **(4 marks)**

d) State FOUR methods of proof   **(2 marks)**

e) Find the domain and range of the function.   **(4 marks)**

$$f(x) = \frac{x-1}{x-5} \quad x \neq 5$$

(4 marks)