TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

SMA 2278: ORDINARY DIFFERENTIAL EQUATIONS

END OF SEMESTER EXAMINATION
SERIES: AUGUST 2013
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) Explain what is meant by the following phrases:
- **(i)** Degree of a differential equation **(1 mark)**
- **(ii)** Complete solution of a differential equation **(1 mark)**
- **(iii)** Exact differential equation **(1 mark)**

$$\frac{dy}{dx} = \frac{y^2 - 4}{x}$$

b) Solve the differential equation **(6 marks)**

c) An object moves with simple harmonic motion on the x-axis. Initially it is located at a distance 45m away from the origin when t = 0 and has velocity V= 13m/s and decelerating at 90ms/s^2 directed towards the origin 0. Find the equation of the position at any time t. **(6 marks)**

d) Using the D-operator method, find the particular solution for the initial value problem.

$$y''-2y'-3y=0$$

if y(0) = 0 and $y^1(0)$ = -4 **(6 marks)**

e) Find the power series solution of the differential equation.

$$\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2+2\right)y=0$$

in powers of x to determine its general solution. **(9 marks)**

SECTION B (Answer any TWO questions from this section)

Question Two

a) Find the inverse Laplace transform:

$$f(s)=\frac{3s+5}{s^2-4}$$

(4 marks)

$$y''-y'=e^x$$

b) By reduction of order solve: **(7 marks)**

$$\frac{dy}{dx}=\frac{x+y-3}{x-y-1}$$

c) Solve the linear fractional equation to obtain the general solution. **(9 marks)**

Question Three

$$(x+y)dx+(3x+3y-4)dy=0$$

a) Solve **(9 marks)**

b) Solve the equation:

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}=5x$$

using the method of reduction of order to find the complementary solution and hence the particular solution **(11 marks)**

Question Four

$$\frac{dy}{dx} + y = e^x$$

a) Find the general solution of: **(4 marks)**

$$\left(D^2 + 1\right)y = \tan x$$

b) Find the particular integral of **(9 marks)**

c) The initial temperature of a body is 53ºC and after 7 minutes its temperature is 43ºC. From Newton's Law of cooling it is known that the rate of cooling of a body is proportional to the temperature difference between the body and its surrounding room temperature. Use this to predict the temperature of the body after a further 7 minutes given that the room temperature was constant at 22ºC. **(7 marks)**

Question Five

a) Using the D-operator method. Find the general solution to:

$$\left(D^2 + 3D - 4\right)y = \sin 2x$$

(6 marks)

b) The velocity of a particle moving along the x-axis is proportional to x. At time t = 0 the particle is located at x = 3 and at t= 12sec it is at x = 6. Find x when t = 4 sec **(9 marks)**

$$\frac{dx}{dt} - 2x = 4$$

c) Solve given t = 0 and x = 1 **(5 marks)**