TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL & AUTOMOTIVE ENGINEERING

SMA 2271: DIFFERENTIAL EQUATIONS

END OF SEMESTER EXAMINATION
SERIES: AUGUST 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Find the Laplace inverse transform of $\dfrac{S+2}{S^2-4S+3}$ **(5 marks)**

b) Solve the differential equation $(3x^2+4xy)dx+(2x^2+2y)dy=0$ **(6 marks)**

c) Determine a general solution of the differential equation $\dfrac{d^2y}{dx^2}+14\dfrac{dy}{dx}+49y=4e^{5x}$ **(6 marks)**

d) Using the D-operator method, find the particular solution $\left(D^2-2D-3\right)y=0$ if $y(0)=0$ and $y'(0)=-4$

(5 marks)

e) Identify all regular singular points of $\left(x^3-3x^2+2x\right)\frac{d^2y}{dx^2}+(x-2)x\frac{dy}{dx}+4x^2y=0$ **(5 marks)**

f) Show that the equation $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0$ has two linearly independent solution of the form e^{ax}.

(3 marks)

Question Two

a) Use Laplace transform to solve $\frac{dx}{dt}+2x=4e^{3t}$ at t = 0 if x = 1 **(5 marks)**

b) An electric circuit has a constant electromotive force E = 40V, a resistor of 10Ω and an inductance 0.2 Henry, with initial current 0 at t = 0 and a differential equation $\frac{Ldi}{dt}+Ri=E;$ determine the steady current after a long time. **(6 marks)**

c) Find the particular integral of $\left(D^2+1\right)y=\tan x$ **(9 marks)**

Question Three

a) By reduction of order solve $y''+2y'=3x$ to find the complementary solution hence the complete solution. **(9 marks)**

b) The initial temperature of a body is 53ºC and after 5 minutes its temperature is 45ºC from Newton's law of cooling it is known that the rate of cooling of a body is proportional to the temperature difference between the body and its surrounding room temperature. use this to predict the 5 minutes given that the room temperature was constant at 21ºC **(7 marks)**

c) Find the Laplace transform of $e^{-3t}\left(2\cos 5t-3\sin 3t\right)$ **(4 marks)**

Question Four

a) Use Bernoulli's method to solve $\frac{dy}{dx}-\frac{1}{2}\left(1+\frac{1}{x}\right)y=\frac{-3y^3}{x}$ **(6 marks)**

$$\frac{dy}{dx}=\frac{x+y-2}{x-y+4}$$

b) Solve the differential equation **(7 marks)**

$$\frac{dy}{dx}=\frac{xy}{x^2+y^2}$$

c) Show that $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ is a homogeneous function in x and y hence using the substitution $y=Vx$ solve the differential equation by separation of variables method. **(7 marks)**

Question Five

a) Show that $(3xy^4+x)dx+(6x^2y^3-2y^2+7)dy=0$ is an exact differential and find its general solution. **(6 marks)**

b) Use the D-Operator method to find the complete solution if

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=6e^{3x}+7e^{-2x}-\log 2$$

(7 marks)

c) Solve the differential equation **(7 marks)**

$$(x+y)dx+(3x+3y-4)dy=0$$