MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

DEPARTMENT OF MATHEMATICS AND PHYSICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN CIVIL ENGINEERING.

SMA 2273: APPLIED MATHEMATICS

DECEMBER : 2011 TIME: 2 HOURS

INSTRUCTIONS. Attempt Question One and any other Two Questions.

QUESTION ONE (30 marks) compulsory.

a) Explain the following terms as used in applied mathematics:
   i) Trajectory 1 mark
   ii) Coplanar forces 1 mark
   iii) Range 1 mark
   iv) Concurrent forces 1 mark

b) A particle is moving along a curve defined by the parametric equations x=2cos3t, y=2sin3t and $z = 4t^2$, find:
   i) the velocity at any time t. 2 marks
   ii) the magnitude of acceleration at time t=0s. 3 marks

c) An aeroplane moves in a northwesterly direction at 125 km/hr relative to the ground due to the fact that there is a westerly wind of 50km/hr relative to the ground. Determine how fast and in what direction the plane would have traveled if there was no wind. 4 marks

d) If the time of oscillation T for a bob of mass m in a simple pendulum of length L is written as $T = Am^x l^y g^z$, where A, x, y and z are constants, find by considering dimensions the values of x, y and z. 5 marks

e) A bullet of mass 30g is fired horizontally into a small block of wood of mass 8 kg which is suspended by a string 2m long. The bullet remains embedded in the wood and the block rises until the string makes an angle of 30^0 with the vertical. Find the velocity of the bullet before impact. 6 marks

f) A particle is projected from a point 0 with an initial velocity of 50 m/s in a direction making an angle α with the vertical. At the same instant a particle is projected vertically downwards with the same speed from a point in the plane of the line of flight 100 metres horizontally and 200 metres vertically

from 0. If the two collide find α^0 in the upper register and the time of flight to the point of impact. 6 marks

QUESTION TWO (20 MARKS)

a) Find the work done in moving a particle in moving a particle once around a circle C in the xy –plane, If the particle has centre at the origin and radius 3 while the force field is given by

$\vec{F} = (2x - y + z)\hat{i} - (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$

. 6 marks

b) ABCDEF is a regular hexagon. The forces of magnitudes 3F, 4F, 2F, 6F act along $\vec{AB}, \vec{AC}, \vec{EA}, \vec{AF}$ respectively. Find the magnitude and direction of the resultant force. 6 marks

c) Find the constant force needed to accelerate a mass of 40kgs from a velocity of $\vec{v_1} = 4\hat{i} - 5\hat{j} + 3\hat{k}$ to the velocity $\vec{v_2} = 8\hat{i} + 3\hat{j} - 5\hat{k}$ in 20sec, what is the magnitude of the force and the work done on the body in the 20 seconds. 8 marks

QUESTION THREE (20 MARKS)

a) A particle of mass 2 kg rests on the surface of a rough plane inclined at 30^0 to the horizontal, it is connected by a light inelastic string passing over a light smooth pulley at the top of the plane to a particle of mass 3 kg which is hanging freely. If the coefficient of friction between the 2 kg mass and the plane is 0.333 find:

i) The acceleration of the system when it is released from rest. 5 marks

ii) Tension in the string. 2 marks

iii) Force exerted by the string on the pulley. 3 marks

b) A force of magnitude 80N acts along the positive x-axis and another force of magnitude 50N is inclined at an angle of 120^0 to this axes. Find their resultant force; stating the magnitude and direction. 4 marks

c) A particle of mass 3 units moves along a space curve defined by $\vec{r} = (4t^2 - t^3)\hat{i} - 5t\hat{j} + (t^4 - 2)\hat{k}$

find i) the momentum. 3marks

ii) force acting on it at time t=2. 3 marks

QUESTION FOUR (20 MARKS)

a) A cyclist moves against a resistance to motion which is proportional to his speed. At a power output of 75W he has a maximum speed of 5m/s on a level road. If the cyclist and bicycle weigh 800N, find the maximum speed he reaches when travelling down a hill inclined at $\theta = \sin^{-1}\frac{1}{40}$ to the horizontal when working at the rate of 25W. 5 marks

b) A particle moves along a path whose parametric equations are $x = 5e^{-2t}$, $y = 4\cos 3t$ and $z = 2\sin 3t$ where t is the time. Find the magnitude of velocity and acceleration at $t = \frac{\pi}{3}$. 6 marks

c) A uniform ladder 5m long weighing 400N rests on a rough horizontal ground and against a smooth vertical wall. If its inclined at 30^0 to the vertical find the normal reaction of the ladder on the ground and the wall. 5 marks

d) A particle is moved along the x-axis by a force P given by $P = \frac{200}{x^2}$ from x=1 to x=6. Calculate the work done assuming the force is in Newton's and displacement in metres. 4 marks

QUESTION FIVE (20 MARKS)

a) A heavy non –uniform plank XY whose weight is 200N rests in a horizontal position on vertical supports at X and Y. The length of the plank is 6m and the centre of gravity is 2.5m from X. find the force exerted by each support. 3 marks

b) A particle of unit mass moves along a curve in a force field given by

$$\vec{F} = (6t-8)\hat{i} - 60t^3\hat{j} + (20t^3 + 36t^2)\hat{k}$$

where t is the time. If its initial position and velocity are

$$\vec{r_o} = 2\hat{i} - 3\hat{k} \quad and \quad \vec{v_o} = 5\hat{i} + 4\hat{j}$$

given respectively by find:

i) The position, velocity, acceleration and momentum of the particle at time t=2 sec. 8 marks

ii) the kinetic energy at t=2 sec. 2 marks

iii) work done from t=0 to t=2 sec. 2 marks

c) Show that the range of a projectile with initial velocity u projected at an angle α^0 to the horizontal is given by $R = \frac{u^2 \sin 2\alpha}{g}$ hence show that the range is a maximum of $\alpha = \frac{\pi}{4}$.

5 marks

THE END