TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE MECHANICAL ENGINEERING
BACHELOR OF TECHNOLOGY IN RENEWABLE ENERGY
BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS
(BSME/BTRE/BTAP)

SMA 2270/SMA 2277/AMA 4209: CALCULUS III

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

$$z = x^3y + x^2y^2 + xy^3$$

a) Find the total differential if **(3 marks)**

b) Determine whether the following sequence in monotonic or not and state the upper bound.

$$|a_n| = \frac{2n}{1+n}$$

(4 marks)

c) Use geometric series to express 0.4166…. as a ratio of two integers **(4 marks)**

$$\lim_{x\to 1}\left\{\frac{1}{\ln x}-\frac{1}{x-1}\right\}$$

d) Evaluate **(4 marks)**

$$\sin 45^o=\frac{1}{\sqrt{2}} \qquad \cos 45^o=\frac{1}{\sqrt{2}}$$

e) Given and . Approximate sin ^{44o} by use of a Taylor's series expansion up to the term containing x^3 **(5 marks)**

$$f(x)=\frac{k}{1+x^2} \qquad (-\infty,\infty)$$

f) The probability density function has the area under the curve on the interval equal to 1, determine the value of K **(6 marks)**

$$\int_2^4\int_1^2\left(x^2+y^2\right)dy\ dx$$

g) Evaluate **(4 marks)**

Question Two

a) Find the moments and centre of mass of the system of objects that have masses 3kg, 4kg and 8kg at the points (-1, 1), (2, -1) and (3, 2) **(4 marks)**

b) A ball is dropped from a height 6m and begins bouncing the height of each bounce is ¾ the previous height. Find the total distance travelled by the ball before it rests **(4 marks)**

c) Find the sum to infinity of the series:

$$\sum_{n=1}^{\infty}\left\{\frac{1}{n}-\frac{1}{n+1}\right\}$$

(5 marks)

$$f(x)+\frac{4}{x}=5$$

d) satisfies the hypothesis of the mean value theorem in the interval (1, 4). Determine the value of C **(4 marks)**

$$r=1+\sin\theta$$

e) Sketch the graph of **(3 marks)**

Question Three

$$\frac{\partial f}{\partial x} \qquad \frac{\partial f}{\partial y} \qquad f(x,y)=\sqrt{9-x^2-y^2}$$

a) Determine and at point (1, 2) for **(4 marks)**

$$f(x)=\cos x$$

b) Find the Maclaurin's polynomial P_0, P_2, P_4 and P_6 for .Use P_6 to approximate the value at $\cos(0.1)$

(5 marks)

c) Evaluate the double integral of $f(x,y) = xy^2 + x^2$ over the region bounded by the curves $y = x^2$ and $x = y^2$ **(4 mark)**

d) Prove that the integral $\int_1^\infty \frac{1+e^{-x}}{x}\,dx$ is divergent **(3 marks)**

e) Evaluate $\lim_{x\to 1} arc\ \sin\left\{\frac{1-\sqrt{x}}{1-x}\right\}$ **(4 marks)**

Question Four

a) Test the series for convergence or divergence $\sum_{n=1}^{\infty}\left\{\frac{n^2-1}{n^2+n}\right\}$ **(3 marks)**

b) Given that $f(x) = x^3 - 12x$ satisfies the Roller's theorem on the interval $0 \le x \le 2\sqrt{3}$. Find the value of C **(3 marks)**

c) Find the length of the one arch of the cycloid $x = r(\theta - \sin\theta)$ and $y = r(1-\cos\theta)$ **(5 marks)**

d) Show that $Z_{xy} = Z_{yx}$ for the function Z given by $Z(x,y) = 2x^2 - 3xy + 4y^2$ **(4 marks)**

e) Find the radius of convergence of the function $f(x) = e^x$ **(5 marks)**

Question Five

a) Find the rectangular coordinates corresponding to the polar coordinates $\left(2, \frac{2\pi}{3}\right)$ **(3 marks)**

b) Find $\frac{dz}{dr}$ and $\frac{dz}{ds}$ given that $z = x^2 + xy + y^2$ where x = 2r + s and y = r – 2s **(4 marks)**

c) Evaluate the $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ **(5 marks)**

$$\lim_{x\to\infty}\left\{\frac{3x^2-x-2}{5x^2+4x+1}\right\}$$

d) Find the value of **(3 marks)**

e) Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy – plane bounded by the line y = 2x and the parabola $y = x^2$ **(5 marks)**