TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BSME/BEME/BSCE/BEBC/BSEE/BMCS

SMA 2277/SMA 2270/ AMA 4209: CALCULUS III

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2013
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FOUR** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Evaluate $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ **(3 marks)**

b) Given $\sin 45^o = \frac{1}{\sqrt{2}}$ and $\cos 45^o = \frac{1}{\sqrt{2}}$ approximate $\sin 44^o$ using the Taylor's series expansion up to x^3 **(3 marks)**

c) Determine $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$ as functions of r and s if $f(x,y,z) = x + 2y + z^2$ and $x = \frac{r}{s}, y = r^2 + \ln$ S, while $z = 2r$

(4 marks)

d) Use geometric series to express $0.08\dot{0}\dot{8}$ as a ratio of two integers **(4 marks)**

e) Evaluate the improper integral:

$$\int_0^2 \frac{dx}{\sqrt{4-x^2}}$$

(4 marks)

f) Find the area enclosed by the curve $r = 1 + \sin\theta$ between $\theta = \frac{\pi}{4}$ and $\theta = \frac{\pi}{4}$ (6 marks)

g) Find the sum of the series $\sum_{n=0}^{\infty} \frac{3}{2^n}$ **(3 marks)**

Question Two

a) Find the value and state if this improper integral is convergent $\int_{-\infty}^{\infty} xe^{-x^2} dx$ **(6 marks)**

b) Evaluate $\int \cos^4 x dx$ **(3 marks)**

c) Find a number C that satisfies the conclusion of the mean value theorem for:

$f(x) = x + \frac{1}{x}$ on $\left(1/2, 2\right)$ **(4 marks)**

d) Prove that $\frac{1}{2} - \frac{2}{2^2} + \frac{3}{2^3} - \frac{4}{2^4} .... = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2^n}$ converges absolutely **(7 marks)**

Question Three

a) Show that the series:

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

is convergent and find its sum **(4 marks)**

$f(x,y)=\frac{xy^2}{x^2+y^4}$ $\lim_{(x,y)\to(0,0)} f(x,y)$

b) If does exist **(4 marks)**

$z=x^2+y^2$

c) Find the area of the part of the paraboloid that lies under the plane z = 9 **(5 marks)**

d) Evaluate the integral **(3 marks)**

$\int(x\ln x)dx$

$\int_1^9\left(\frac{2t^2+t^2\sqrt{t}-1}{t^2}\right)dt$

e) Evaluate **(4 marks)**

Question Four

$z=x^2+y^2$

a) Find the volume of the region that lies inside the region and below the plane z = 16 **(8 marks)**

$\{a_n\}$ $\{a_n\}=\frac{-2}{1},\frac{8}{2},\frac{-26}{6},\frac{80}{24},\frac{-242}{120}$

b) Find the nth term of a sequence whose first five term are (3 marks)

c) State the Rolle's theorem hence verify using Rolle's theorem that its satisfied by the function $f(x)=5x^2+7x-6$ in (-3, 4) **(4 marks)**

$f(x)=\ln x$

d) Find the Taylor's polynomial P_4 for contained at x = 1.1 **(5 marks)**

Question Five

$\int_{-2}^{3}\int_{0}^{\pi/2} x\cos y\,dx\,dy$

a) Evaluate **(5 marks)**

$x(t)=r\cos t$ $y(t)=r\sin t$ $0\le t\le 2\pi$

b) Find the length of the curve defined parametrically by and for **(4 marks)**

$1<a<b$ $\lim_{x\to\infty}\left(a^x+b^x\right)^{1/x}$

c) Given that find **(6 marks)**

d) Determine the surface are generated by revolving the circle $x^2 + y^2 = 9$ about the x-axis from x = -2 to x = 2 **(5 marks)**