



# TECHNICAL UNIVERSITY OF MOMBASA

## Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

**BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE**  
**BACHELOR OF SCIENCE IN STATISTICS & COMPUTER SCIENCE**  
**BACHELOR OF SCIENCE IN INFORMATION COMMUNICATION TECHNOLOGY**  
**(BMCS/BSSC/BTIT)**

AMA 4101/SMA 2102: CALCULUS I

**END OF SEMESTER EXAMINATION**

SERIES: APRIL 2015

**TIME ALLOWED: 2 HOURS**

### Instructions to Candidates:

You should have the following for this examination

- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

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### **Question One (Compulsory)**

$$y = \frac{3}{x}$$

a) Differentiate  $y = \frac{3}{x}$  from first principles **(3 marks)**

b) Determine the equation of the tangent and normal to the curve  $y = x^3$  at the point (1, 1) **(5 marks)**

c) Plot the graph of  $y = x^2 - 4x$  between  $x = -2$  and  $x = 3$ . Hence or otherwise find the area enclosed by the curve and the x axis **(6 marks)**

$$\frac{dy}{dx} \quad y = 3 \sin 3t$$

$$x = 2 \cos t$$

d) Find  $\frac{dy}{dx}$  given that (3 marks)

$$\lim_{x \rightarrow 1} \frac{2x^2 + 2x - 4}{x - 1} = 6$$

e) Prove that (4 marks)

$$g(x) = \begin{cases} x^3, & x \geq 2 \\ x^2 + 4, & x < 2 \end{cases}$$

f) Check for continuity at  $x = 2$  for the function (3 marks)

$$\frac{dy}{dx} \quad 5y^2 + \sin y = x^2$$

g) By implicit differentiation, find  $\frac{dy}{dx}$  if (3 marks)

h) The side of a square is 5cm. Find the increase in area of the square when the side expands by 0.01cm (3 marks)

### Question Two

$$f(x) = x^3 + 3x^2 - 9x - 13$$

a) Use the second derivative test to find the local extrema of (7 marks)

$$\frac{d}{dx}(\sin x) = \cos x$$

b) Show that  $\frac{d}{dx}(\sin x) = \cos x$  using first principles (6 marks)

c) Evaluate:

$$\lim_{x \rightarrow 2} (2x^2 - x + 1)$$

(i)

$$\lim_{x \rightarrow \infty} \frac{6x - 1}{2x + 1}$$

(ii)

$$\lim_{x \rightarrow -3} \frac{3x + 9}{x^2 - 9}$$

(iii)

marks)

(7

### Question Three

$$f(x) = x^3 \quad g(x) = 1 + x^2 \quad (f \circ g)(x) \quad (g \circ f)(x)$$

a) Given  $f(x) = x^3$  and  $g(x) = 1 + x^2$  Find  $(f \circ g)(x)$  and  $(g \circ f)(x)$  (6 marks)

$$y = -x^3 + 6x^2$$

b) Find the points on the graph  $y = -x^3 + 6x^2$  at which the tangent has slope 6 (8 marks)

c) It is estimated that  $x$  months from now, the population of a certain community will be

$$P(x) = x^2 + 20x + 8000$$

(i) At what rate will the population be changing with respect to time 15 months from now (3 marks)

(ii) By how much will the population actively change during the 16<sup>th</sup> month (3 marks)

### Question Four

a) Evaluate the following limits:

$$\lim_{x \rightarrow 1} \frac{\sqrt{x-1}}{x-1}$$

(i) (4 marks)

$$\lim_{x \rightarrow \infty} \frac{x^2}{1+x+2x^2}$$

(ii) (3 marks)

$$y = \frac{x^2 \sin x}{\cos 2x}$$

b) Find the derivative of the function using logarithmic differentiation (5 marks)

c) An object travels in such a way that distance S (in metres) from the starting point is a function of time

$$s(t) = 10t^2$$

t (in hours) as follows

(i) Find the average velocity between the time  $t = 2$  and  $t = 5$  (3 marks)

(ii) Find the instantaneous velocity when  $t = 4$  (3 marks)

(iii) Find the acceleration when  $t = 4$  (2 marks)

### Question Five

a) Given the parametric equation of a function

$$y = 3\sin \theta - \sin^3 \theta$$

$$x = \cos^3 \theta$$

evaluate  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$

(8 marks)

b) Use a tangent line approximation to estimate the value of

$$\sqrt[3]{1.1}$$

(6 marks)

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

c) Prove that

(6 marks)