TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS
BACHELOR OF TECHNOLOGY IN RENEWABLE ENERGY
(BTAP/BTRE)

AMA 4109: CALCULUS FOR TECHNOLOGISTS I

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Evaluate $\lim_{x\to\infty}\left\{\frac{3x^2 - x - 2}{5x^2 + 4x + 1}\right\}$ **(4 marks)**

b) Sketch the graph of the function f(x) given by $f(x) = x(x-3)^2$ (5 marks)

c) If $x^2 + y^2 = 25$ find dy/dx hence determine the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point (3, 4) **(6 marks)**

d) Prove that if f and g are both differentiable then $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

(5 marks)

e) Evaluate the triple integral: $\int_0^1 \int_0^{x^2} \int_{xy}^{x+y} (xyz\ )dz\ dy\ dx$ **(4 marks)**

f) If $z = 2x^2 - 3xy + 4y^2$. Show that $Z_{xy} = Z_{yx}$ **(6 marks)**

Question Two

a) Evaluate $\lim_{x \to 0} \left\{ \frac{e^{2x} - 1}{x} \right\}$ **(5 marks)**

b) Work out the double integral: $\int_1^2 \int_3^4 (y - x) dydx$ **(4 marks)**

c) Air is being pumped into a spherical balloon so that its volume increases at a rate of 100cm^3/s. How fast is the radius r of the balloon increasing when the diameter is 50cm? **(7 marks)**

d) Given that $f(x) = 5 - \frac{4}{x}$, find C in the open interval (1, 4) using the mean value theorem

(4 marks)

Question Three

a) If $f(x) = x^3 - x$, find a formula for f'(x) by first principle **(4 marks)**

b) Differentiate each of the following:

(i) $y = \frac{1}{x^3}$ **(1 marks)**

(ii) $y = e^x - x$ **(2 marks)**

(iii) $y = \frac{\sec x}{1 + \tan x}$ **(3 marks)**

c) Find the volume of a solid that is obtained when the region under the curve $y = \sqrt{x}$ over the interval (1, 4) is revolved about the x-axis **(5 marks)**

d) Find the point on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal **(5 marks)**

Question Four

a) State THREE ways through which a function fails to be differentiable **(3 marks)**

b) Determine the integrals given by:

(i) $\int \frac{1}{x^2}\, dx$ **(2 marks)**

(ii) $\int \cos 3x \; dx$ **(1 mark)**

(iii) $\int_0^3 \left(x^3 - 6x\right)\, dx$ **(3 marks)**

c) Find the area of the region bounded by y = x + 6 and below by $y = x^2$ and on the sides by x = 0 and x = 2 **(5 marks)**

d) Find the length of one arch of the cycloid $x = r(\theta - \sin\theta) \quad y = r(s - \cos\theta)$ with $0 \le \theta \le 2\pi$ **(6 marks)**

Question Five

a) Discuss the stationary points in the curve $y = x^4 - 4x^3$ **(4 marks)**

b) Suppose an area of farmland along a straight stone wall is to be fenced. There are 400m of fencing available. What is the greatest rectangular area that can be enclosed? Note: Part of stone does not need fencing. **(4 marks)**

c) A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $V(o) = -6cm/s$ and in initial displacement in s(o) = 9cm. Find its position function s(t) **(5 marks)**

d) The position of a particle moving along the x-axis is given by the function $x(t) = 3t^4 - 32t^3 + 114t^2 - 144t + 40, \quad 0 \le t \le 5$ where x is measured in metres and t measured in seconds.

(i) What are its velocity and speed at t = ½ seconds **(2 marks)**

(ii) When is its acceleration increasing **(2 marks)**

(iii) Is the particle speeding up or slowing down at t = 2 seconds? **(2 marks)**

(iv) What is the maximum distance the particle ever attains from the origin **(1 mark)**