



**TECHNICAL UNIVERSITY OF MOMBASA**  
**Faculty of Applied & Health**  
**Sciences**

DEPARTMENT OF MATHEMATICS & PHYSICS

**UNIVERSITY EXAMINATION FOR:**  
BACHELOR OF TECHNOLOGY IN ANALYTICAL CHEMISTRY (BTAC)

AMA 4109: CALCULUS FOR SCIENCE

**END OF SEMESTER EXAMINATION**

SERIES: APRIL 2014

**TIME ALLOWED: 2 HOURS**

**Instructions to Candidates:**

You should have the following for this examination

- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

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**Question One (Compulsory)**

a) Define the following terms:

- (i) Function (2 marks)  
(ii) Monotonic function (2 marks)

b) Given that  $f(x) = 2x - 1$  and  $g(x) = 3 - 5x$  find  $f \circ g(x)$  and hence  $f \circ g(x)^{-1}$  (4 marks)

c) Find  $\frac{dy}{dx}$  given that  $u = x^2$  and  $y = \cos u$  (4 marks)

d) An open box with a square bottom is to be cut from a piece of cardboard 10 feet by 10 feet by cutting out the corners and folding the sides up. Find the dimensions that will result in the largest volume. **(5 marks)**

e) Use partial fractions to evaluate the integral:

$$\int \frac{2x dx}{x^2 - x - 2}$$

$$f(x) = 6x^5 + 33x^4 - 3x^3 + 100$$

f) Determine all the critical points of the function **(5 marks)**

$$\int_0^2 10x + 10 dx$$

g) Evaluate **(3 marks)**

**Question Two**

$$\frac{dy}{dx} \quad y = \frac{u-1}{u+1} \quad u = x^2$$

a) Find  $\frac{dy}{dx}$  given that  $y = \frac{u-1}{u+1}$  and  $u = x^2$  **(5 marks)**

b) A man is on an island which is 4 miles from the nearest point on a straight shoreline. He wishes to go to a house which is 12 miles from the nearest point. If he rows at 3 miles per hour and runs at 5 miles per hour, find the shortest time to reach the house. **(6 marks)**

c) Evaluate the following limits.

(i)  $\lim_{x \rightarrow 3} \frac{x^3 - 2x + 4}{x^2 + 1}$  **(3 marks)**

(ii)  $\lim_{x \rightarrow \infty} \frac{2x^4 - x^2 + 8x}{-5x^4 + 7}$  **(3 marks)**

$$\lim_{x \rightarrow 0} \frac{|x|}{x}$$

d) Show that  $\lim_{x \rightarrow 0} \frac{|x|}{x}$  does not exist. **(3 marks)**

**Question Three**

a) Evaluate the following integrals:

(i)  $\int x^2 \sin 10x \, dx$  **(5 marks)**

(ii)  $\int \frac{dx}{1 + \sqrt{x}}$  using the substitution  $u^2 = x$  **(4 marks)**

b) Find the area of the region bounded by  $y = x^2$  and  $y = \sqrt{x}$  **(6 marks)**

- c) Find  $f'(x)$  using first principles, given that  $f(x) = x^3 - 12x$  **(5 marks)**

**Question Four**

$$f(x) = \begin{cases} x^2 & x \leq 1 \\ \frac{1}{2}x + \frac{1}{2} & x > 1 \end{cases}$$

- a) (i) Show that  $f(x)$  is not differentiable at  $x = 1$  **(3 marks)**  
(ii) Define the term differentiable function. **(3 marks)**

$$y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8$$

- b) Classify all the critical point of  $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8$  hence sketch the curve. **(10 marks)**

$$\frac{d^2y}{dx^2} \quad y = \sqrt{2 - 3x^2}$$

- c) Find  $\frac{d^2y}{dx^2}$  given  $y = \sqrt{2 - 3x^2}$  **(4 marks)**

**Question Five**

- a) Air is being pumped into a spherical balloon so that its volume increases at a rate of  $100\text{cm}^3/\text{s}$ . How fast is the radius of the balloon increasing when the diameter is  $50\text{cm}$ ? **(6 marks)**

$$y = \sqrt{x}$$

- b) Find the volume of the solid obtained by rotating about the x-axis the region under the curve from  $x = 0$  to  $x = 1$ . **(6 marks)**

$$\frac{dy}{dx}$$

- c) Find

$$y = \frac{3x - 1}{5x + 2}$$

- (i) **(4 marks)**

$$y = (x^3 + 1)(3x^5 + 2x - 1)$$

- (ii) **(4 marks)**