TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE

AMA 4321: ANALYTICAL APPLIED MATHEMATICS

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

<u>Instructions to Candidates:</u>
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FOUR** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **TWO** printed pages

Question One (Compulsory)

$$n = 2\int_0^\alpha e^{-t2} t^{\;2n-1} dt$$

a) Show that **(5 marks)**

$$B(m,n) = \frac{m \quad n}{m+n}$$

b) Establish the relation **(5 marks)**

$$H(t) = \begin{cases} t+1 & if\; 0 \le t \le 2 \\ 3 & if \;\; f > 2 \end{cases}$$

c) Find the Laplace transform of **(5 marks)**

$$\left(D^2 + 2DD^1 - 8D^{12}\right)z = \sqrt{2x+3y}$$

d) Solve **(6 marks)**

e) Express $1-x+x^2$ in the form of the Legendre polynomial **(3 marks)**

f) Form a partial differential equation from $z = f(x+ct)+g(x-ct)$ **(3 marks)**

g) Evaluate $\dfrac{dx}{\sqrt[3]{1-x^3}}$ **(3 marks)**

Question Two

a) Use the Power series method to solve the differential equation about the ordinary point x = 0 **(10 marks)**

$$\left(1-x^2\right)y''-6xy'-4y=0$$

b) Solve the differential equation below using the Laplace transform:

$$y''(t)+6y'(t)+9y(t)=t^2e\ \ ty(0)=2y'(0)=6$$

(10 marks)

Question Three

a) If $J_n^{(x)}$ is the Bessel function of order n, prove that:

(i) $J_{1/2}^{(x)}=\sqrt{\dfrac{2}{\pi x}}\sin x$ **(5 marks)**

(ii) $\dfrac{d}{dx}\left(x^{-n}J_n^{(x)}\right)=-x^{-n}J_{n+1}^{(x)}$ **(3 marks)**

b) Find the Fourier series of the function:

$f(x)=|x|$ in the interval $-\pi<x<\pi$ **(7 marks)**

c) Find the singular, regular singular and irregular singular point of the differential equation $2x^2\,{dy^2}/{dx^2}-x\,{dy}/{dx}+(x-5)y=0$

(5 marks)

Question Four

a) Prove the Legendre duplication formula:

$$2n=2^{2n-1}\frac{n\quad n+1}{\pi}$$

(10 marks)

$$2Fi\left(\frac{-n}{2},\frac{-n}{2}+{}^{1}\!/_{2},b+{}^{1}\!/_{2},1\right)=\frac{2^{n}(b)_{n}}{(2b)_{n}}$$

b) Show that **(10 marks)**

Question Five

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

a) Form a partial differential equation from **(8 marks)**

$$pz-qz=z^2+(x+y)^2$$

b) Solve the partial differential equation: **(6 marks)**

$$\left(D^2-2DD''+D'+2D+2D'+1\right)Z=0$$

c) Find the general solution of: **(6 marks)**