



THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF TECHNOLOGY IN INFORMATION & COMMUNICATION TECHNOLOGY (YR 1 SEM 1)

AMA 4103: CALCULUS I

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: OCTOBER 2011

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Answer booklet

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

This paper consist of **THREE** printed pages

Question One (30 marks)

a) Define the following terms

(i) A Surjective function

(ii) A Bijective function

(4 marks)

b) Let $g: \mathfrak{R} \rightarrow \mathfrak{R}$ be defined by $h(x) = \begin{cases} x^2 - 4x & x \geq 3 \\ x + 3 & x < 3 \end{cases}$ find $h(5)$, $h(0)$, $h(-3)$ (3 marks)

c) Find $\frac{dy}{dx}$ for the following functions
 $y = \sin(\sin(x^2))$

(i)

(4 marks)

(ii) $y = x^2 \tan x$ (2 marks)

d) Evaluate the following limits

(i)
$$\lim_{x \rightarrow 8} \frac{x^{\frac{2}{3}} + 3\sqrt{x}}{4 - \frac{16}{x}}$$
 (4 marks)

(ii)
$$\lim_{x \rightarrow -2} \frac{x+2}{x^2+x-2}$$
 (3 marks)

e) Find the derivative of $y = \sqrt{x+2}$ by the first principles (5 marks)

f) Evaluate the following integrals $\int \sqrt{1+x^2} 2x dx$ (5 marks)

Question Two (20 marks)

a) Define continuity of a function at a point $x = b$ (4 marks)

$$f(x) = \frac{x^2 + x - 6}{x^2 - 4}$$

b) Define $f(2)$ in a way that extends to be continuous at $x = 2$ (6 marks)
 $y = x^2 + x$

c) Find the equation of both lines through (2, -3) that are tangents to the curve (10 marks)

Question Three (20 marks)

a) Let $f(x) = 2x + 1$ and $g(x) = \frac{x}{3}$. Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ (8 marks)

b) Given that $f(0) = 8$, $f'(0) = 3, g(0) = 5, g'(0) = 1$, find the derivative of $F(x)$ at $x = 0$ where $F(x) = \frac{f(x)}{g(x)} + 3x^2 + 4x$ (4 marks)

c) Find the derivative of the following functions

- (i) $x^2 + 2xy + y^2 = 3$ (4 marks)
- (ii) $y = e^{\cos 2x}$ (4 marks)

Question Four (20 marks)

a) Find the value of k for which the following function is continuous

$$f(x) = \begin{cases} x^3 + 2 & x \leq 1 \\ kx + 5 & x > 1 \end{cases}$$

(4 marks)

b) Find the area of the region between the curves $y = x^2$ and $y = 2x - x^2$ (6 marks)

c) Find $\frac{dy}{dx}$ in the following (4 marks)

$$y = \ln\left(\frac{x\sqrt{+5}}{(x-1)^3}\right)$$

d) Use differentials and the function $y = \sqrt[3]{x}$ to approximate $y = \sqrt[3]{126}$ (6 marks)

Question Five (20 marks)

a) Find $\frac{dy}{dx}$ for the following (4 marks)

$$x = 2t^4, y = 6t^2 - 5t$$

b) How fast does the water level drop when a cylindrical tank is drained at the rate of 3 litres/sec? (5 marks)

c) Evaluate $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = 3$. (5 marks)

d) Find the dimensions of a rectangular computer lab with perimeter 100m whose area is as large as possible. Find this maximum area (6 marks)