TECHICAL UNIVERSITY OF MOMBASA

Faculty of Engineering & Technology

DEPARTMENT OF BUILDING & CIVIL ENGINEERING

HIGHER DIPLOMA IN CIVIL ENGINEERING (HDBC)

AMA 3101: CALCULUS

END OF SEMESTER EXAMINATION
SERIES: APRIL 2013
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination

- *Answer Booklet*
- *Mathematical Tables*
- *Scientific Calculator*

This paper consists of **FIVE** questions.

Answer any **THREE** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One

$$\lim_{x\to\infty}\left(\frac{3n-2}{5n+4}\right)$$

a) (i) Evaluate **(3 marks)**

(ii) Determine all the numbers C, which satisfy the conclusions of the main value Theorem for the following function.

$$f(x) = x^3 + 2x^2 - x \quad on \quad [-1,\ 2]$$

(5 marks)

$$\int_0^{\sqrt{x}} \int_0^{e^{x/y}} dy\ dx$$

b) (i) Solve, **(3 marks)**

(ii) Use L'Hospital's Rule to determine limit of the function $f(x) = \frac{\sin x - x}{\tan x - x}$ as $x \to 0$ **(5 marks)**

(iii) Evaluate $\int \tan^n x\ dx$ **(4 marks)**

Question Two

a) (i) Given $f(x) = \sqrt[3]{x}$ approximate the value for $\sqrt[3]{1.1}$ using Taylor's theorem. **(6 marks)**

(ii) A manufacturer needs to make a cylindrical can that will hold 1.5 litres of liquid. Determine the dimensions of the can that will minimize the amount of material used in its construction.

(8 marks)

$$\int_0^{0.6} \int_0^{.0.8} \int_0^{0.3} (4z - 3x - y) dz dy dx$$

b) (i) Evaluate **(4 marks)**

(ii) Determine $\frac{\partial^2 z}{\partial x \partial y}$ of $z(x, y) = e^{3^{x^2}} y$ **(2 marks)**

Question Three

$$\int \frac{x^n}{\sqrt{(a^2 + x^2)}}\ dx$$

a) (i) Evaluate:

(ii) Use the above solution in Q3 (a) (i) to solve,

$$\int \frac{x^5}{\sqrt{(2+x)}}dx$$

(12 marks)

b) Given $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ then find the value of $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ **(8 marks)**

Question Four

a) (i) Find by double integration, the area contained between the two curves, $y^2 = 4x$ and $x2 = 4y$ **(5 marks)**

(ii) Find limit of the function:

$$f(x) = \frac{2x^2 - x - 3}{(x+1)}$$ as $x \to -1$ **(3 marks)**

b) Use Taylor's series to approximate the value of sin 46º correct to six decimal places. **(6 marks)**

c) Given $f(x, y) = e^{x^2} \cdot e^y \cos\left(x - e^{y^3}\right)$. Determine:

(i) $\partial f / \partial x$

(ii) $\partial^2 f / \partial x \partial y$

(iii) $\partial f / \partial y$ **(6 marks)**

Question Five

a) (i) Evaluate $\int_{-\infty}^{+\infty} \frac{4x^3}{\left(1+x^4\right)^2}\, dx$ **(3 marks)**

(ii) Determine whether $\int_1^4 \frac{dx}{(x-2)^{2/3}}$ converges **(2 marks)**

$$f(x) = \sqrt{x-1} \qquad [2,5]$$

b) (i) A bridge described by the function on . The bridge is continuous and oscillates between the parameters 2 and 5.

Show that the bridge observes the mean value theorem hypothesis. **(6 marks)**

(ii) Volume of a metal box is found to be influence by weather. During cold season, the length decreases by 0.008 and width by 0.125. Determine its charge in volume at this season. **(4 marks)**

c) Evaluate:

$$\int_0^1\int_0^1\int_0^1 \left(\frac{2x-y^2}{z}\right)$$

(5 marks)