THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)
Faculty of Engineering and Technology

DEPARTMENT OF BUILDING AND CIVIL ENGINEERING

HIGHER DIPLOMA IN BUILDING & CIVIL ENGINEERING

AMA 3101: CALCULUS III

END OF SEMESTER EXAMINATION

SERIES: DECEMBER 2011

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Answer Booklet*
- *Calculator/Mathematical Tables*

This paper consists of **FIVE** questions
Answer question **ONE (COMPULSORY)** from **SECTION A** and any other **TWO** questions from **SECTION B**
Maximum marks for each part of a question are clearly shown
This paper consists of **FOUR** printed pages

SECTION A (COMPULSORY)

Question 1 (30 marks)

a) Find all the first order partial derivative of the following:

(i) $$f(x,y) = y^7 In(x^2) + \frac{9}{y^3} - \sqrt{x^2}$$

(ii) $$f(x,y,z) = \cos\left\lfloor \tfrac{4}{x} \right\rfloor e^{x^2 y} + 5z$$

(iii) $$z = In(2x + y)$$

(iv) $$x^2 + y^2 + z^2 = xyz^2$$ (15 marks)

b) Evaluate each of the following limits using L'Hospital's Rule:

(i) $$\frac{\tan\ x^2}{2x}$$

(ii) $$\lim_{x \to \pi} \quad \frac{\sin\ x + 2x}{x}$$

(5 marks)

c) Evaluate,

(i) $$\int e^x x^n dx,$$

(ii) Use this expansion to solve, $$\int_0^2 e^x x^5 dx$$ (10 marks)

SECTION B (Answer any TWO questions from this section)

Question 2 (20 marks)

a) (i) Obtain a reduction formula for the intergral, $$\int x^n \cos x\ dx$$

(ii) Use above integral in Q.2 9(a) (i) to solve, $$\int_0^{\pi/2} x^3 \cos x\, dx$$ (8 marks)

b) Determine the average value of each of the following functions on the given intervals;

$$f(t) = t^2 - 5t + 6\cos(\pi t) \; on \left[-1, \frac{5}{2}\right]$$

(i)

$$R(z) = \sin(2z)e^{1-\cos(2z)} \; on [-\pi, \pi]$$

(ii) (4 marks)

c) Evaluate $\int y\, ds,$ over the area of that part of the circle, $x^2 + y^2 = a^2$ contained in the first quadrant. (8 marks)

Question 3 (20 marks)

a) Determine the number, c, that satisfies the Mean Value Theorem for integrals for the function $f(x) = x^2 + 3x + 2$ on the interval (1,4) (4 marks)

$$f(x) = \frac{x-4}{3\sqrt{(x-2)}}, x \neq 4.$$

b) Find limit of the function (4 marks)

c) Use Taylor's series to determine the value of tan 64°, (to 5 decimal places) (8 marks)

$$\int_{-\infty}^{+\infty} \frac{4x^3}{(1+x^4)^2}\, dx$$

d) Check whether the following integral converges or diverges (4 marks)

Question 4 (20 marks)

a) Using $f(x) = \sqrt[3]{x},$ approximate the value for $\sqrt[3]{1.1}$, using Taylor's theorem (8 marks)

b) At 7 p.m, a car is travelling at 50 miles per hour. Ten minutes later, the car has slowed to 30 miles per hour. Show that at some time between 7 and 7:10 the car's acceleration is exactly 120, in units of miles per hours squared. (3 marks)

c) Evaluate $\lim_{x \to 0} \frac{\tan x - x}{\sin x - x}$ (4 marks)

d) A metallic box 4cm length, 3cm wide and 2.5cm high is influenced by temperature change. Find the change in volume when the length is increased by 0.25, width is decreased by 0.11 and height is decreased by 0.25 (5 marks)

Question 5 (20 marks)

$u = e^{r\cos\theta}\cos(r\sin\theta)$.

a) If Determine:

(i) $\dfrac{\partial^2 u}{\partial r \partial \theta}$

(ii) $\dfrac{\partial^2 u}{\partial \theta^2}$ (8 marks)

b) Determine whether the following diverges or converges

(i) $\int_0^1 \dfrac{1}{\sqrt{x}}\,dx$

(ii) $\int_0^1 In\ x\,dx$ (6 marks)

c) Evaluate the following:

(i) $\int\int r\cos\left(e^{(x+y^3)}drdy\right)$

(ii) $\int_1^2\int_0^x \dfrac{1}{x^2}\,dx\ dy$ (6 marks)