THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Engineering and Technology

DEPARTMENT OF BUILDING AND CIVIL ENGINEERING

DIPLOMA IN BUILDING & CIVIL ENGINEERING
DIPLOMA IN CIVIL ENGINEERING & CAD

AMA 2303: CALCULUS IV

END OF SEMESTER EXAMINATION

SERIES: AUGUST/SEPTEMBER 2011

TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination

- *Answer booklet*
- *Calculator*
- *Mathematical table*

This paper consists of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question 1

a) Let $w = f\{x(t), y(t)\}$

$f(x, y) = x^2 y + y^2$

$x(t) = t^2$ and $y(t) = 2t$

Find $\frac{\partial w}{\partial t}$ by use of chain rule (4 marks)

b) Find the critical points of : (5 marks)

$f(x, y) = 3xy - x^3 - y^3$

c) Compute the directional derivative of:

$f(x, y) = x^3 y - 4y^2$ at (-2, 3) in the direction $\vec{u} = \frac{3}{5} i + \frac{4}{5} j$ (4 marks)

d) Apply Green's theorem to evaluate

$\oint_c y^2 dx + x^2 dy$

Over the triangle bounded by $x = 0$, $x + y = 1$ and $y = 0$ (5 marks)

e) Determine if the following integral is convergent or divergent and if it is convergent, find its value

$\int_1^\infty \frac{1}{x} dx$

(4 marks)

f) Find the maximum and minimum values of $f(x, y) = x + 2y$ subject to the constraint $x^2 + y^2 = 1$ (8 marks)

SECTION B (Answer *any TWO questions from this section*)

Question 2

a) Consider the function $f(x, y, z) = x\ y^2 + yz^3 + x\ y^2$

i) Find the gradient vector of *f* at (5, 4, -1)

$$u = \frac{2}{\sqrt{20}} \underset{\sim}{i} - \frac{3}{\sqrt{20}} \underset{\sim}{j} - \frac{3}{20} \underset{\sim}{k}$$

ii) Find the rate of change of f at (4, 5, -1) in the direction (9 marks)

b) A rectangular area adjacent to a stone wall is to be enclosed using a chain –link fence. Because the stone wall forms one side of the rectangle, only the three remaining sides need to be fenced. Taking the width of the area to be x and the length to be y, find the maximum area that can be enclosed by 50 metres of fence. (Use Lagrange multiplier method) (11 marks)

Question 3

Evaluate the integrals below

a) $\int_1^{\infty} \frac{dx}{x^5}$ (5 marks)

b) $\int_0^3 \frac{dx}{\sqrt{3-x}}$ (5 marks)

c) $\int_{-1}^{\infty} e^{-5x}$ (5 marks)

d) $\int_0^1 \int_{2x}^2 x + y \quad dy\, dx$ (5 marks)

Question 4

a) Use the divergence theorem to find the outward flux of

$$F = (y-x)i + (x-y)j + (y-x)k$$

Across the boundary of the cube bounded by the planes $x = \pm 1,\ y = \pm 1$ and $z = \pm 1$ (12 marks)

b) Find the integral of $x^2 + y^2 \text{ over the region } R: 0 \le x \le 2, 1 \le y \le 4, 0 \le z \le 5$ (8 marks)

Question 5

Find the dimension of the box with the largest volume if the total surface area is 64cm^2 (20 marks)

