TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF MATHEMATICS & COMPUTER SCIENCE

AMA 4306: THEORYOF ESTIMATION

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FOUR** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Let Y_1, Y_2 and Y_3 be a random sample from a normal distribution where μ and σ are unknown. Which is a more efficient estimator for μ

$$\hat{u} = \frac{1}{4}Y_1 + \frac{1}{2}Y_2 + \frac{1}{4}Y_3$$

$$\hat{\mu}_2 = \frac{1}{3}Y_1 + \frac{1}{3}Y_2 + \frac{1}{3}Y_3$$

(7 marks)

b) Suppose the random variables X_1, X_2,…X_n denote the number of successes (0 or 1) in each of n independent trials. Where P = P (success occurs at any given trial) is unknown parameters. Then:

$$Px_i(k\ ;p) = p^k(1-p)^{1-k},\quad k = 0.1;\ 0 < p < 1$$

Let x = $x_1 + x_2 + \ldots x_n$ = total number of successes and define $\hat{p} = \frac{x}{n} \cdot \hat{p}$ is unbiased for P. Compare var $\left(\hat{p}\right)$ with the cramer-Rao lower bound for P_{xi}(k;p)? **(8 marks)**

c) Given that $y_1 = 2.3$, $y_2 = 1.9$ and $y_3 = 4.6$ is a random sample from:

$$f_r(y;\theta) = y^3 e^{-y/\theta} / 6\theta^4 \quad y \geq 0$$

Calculate the maximum likelihood estimate for θ (6 marks)

d) Let $g\Theta(\theta|w = w)$ be the posterior distribution for the unknown parameter θ. If the loss function associated with $\hat{\theta}$ is $L\left(\hat{\theta},\theta\right) = \left|\hat{\theta} - \theta\right|$ then the Bayes estimate for θ is the median of $g\Theta(\theta|w = w)$ Give proof to the above statement. **(9 marks)**

Question Two

a) By definition the geometric mean of a set of n numbers is the nth root of their product. Let Y_1 and Y_2 be a random sample of size two from the pdf

$$f_r(y;\theta) = \frac{1}{\theta} e^{-y/\theta}, \quad y > 0$$

where θ is an unknown parameter. Find an unbiased estimator for θ based on the sample's geometric mean $\sqrt{y_1 y_2}$ **(10 marks)**

b) Suppose that 6.5, 9.2, 9.9 and 12.4 constitute a sample of size four from the pdf

$$f_r(y;u) = \frac{1}{\sqrt{2\pi}(0.8)} e^{\frac{-1}{2}\left(\frac{y-\mu}{0.8}\right)^2} \quad -\infty < y < \infty$$

, what values of u are believable in the light of the four data points? (at 955 confidence level) **(10 marks)**

Question Three

a) An experimenter has reason to believe that the pdf describing the variability in a certain type of measurement is the continuous pdf:

$$f_r(y;\theta) = \frac{1}{\theta^2} y e^{-y/\theta} \qquad 0 < y < \infty,\quad 0 < \theta < \infty$$

Five data points have been collected 9.2, 5.6, 18.4, 12.1 and 10.7. Find the maximum likelihood estimate for θ **(9 marks)**

b) Based on the data below, it is hypothesized that x has a Poisson distribution

	Observed
Number of major changes	Frequency
0	237
1	90
2	22
3	7
	356

Confirm that $Px = \frac{e^{-\lambda}\lambda^k}{k!}$ can provide an adequate fit to these 356 observations **(11 marks)**

Question Four

a) Suppose that $y_1 = 0.42$, $y_2 = 0.10$, $y_3 = 0.65$, $y_4 = 0.23$, $y_5 = 0.41$ is a random sample of size four from the pdf:

$$f_r(y;\theta) = \theta y^{\theta-1} \qquad 0 \le y \le 1$$

Find method of moments estimate for θ (11 marks)

b) A random sample of size 10, $x_1 = 1$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$, $x_5 = 0$, $x_6 = 1$, $x_7 = 1$, $x_8 = 0$, $x_9 = 11$ and $x_{10} = 1$; is taken from the probability function:

$$px(k;\theta) = \theta^k(1-\theta)^{1-k}, \; k = 0,1; \; 0 < \theta < 1$$

Find the maximum likelihood estimate for θ **(9 marks)**

Question Five

a) A two parameter gamma pdf $f_r(y;r,\lambda) = \frac{\lambda^r}{(r)} y^{r-1} e^{-\lambda y} \quad , y > 0$ is used to model, the maximum twenty, four-hour precipitation. Use the method of moments to estimate r and λ [use $E(Y) = r/\lambda$ and $E(Y^2) = r(r+1)/\lambda^2)$] for 36 observation given below: **(6 marks)**

31.00	6.31	4.21	3.69	3.73
2.82	4.95	11.60	3.10	3.50

3.98	5.64	4.75	22.22	6.20
4.02	5.51	6.85	7.43	0.67
9.50	13.40	6.25	5.00	
4.50	9.72	3.42	4.58	
11.40	6.47	11.80	4.46	
10.71	10.16	0.80	8.00	

b) (i) Use the data in (a) to get an estimate of the model **(2 marks)**

(ii) Plot a histogram of the data **(6 marks)**

(iii) Superimpose the estimated model on the Histogram and comment on the fit **(6 marks)**