



TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

SMA 2272: STATISTICS

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: OCTOBER 2013

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Answer Booklet*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **FOUR** printed pages

SECTION A (COMPULSORY)

Question One

a) Define the following terms:

- (i) A random variable (1 mark)
- (ii) A probability distribution (1 mark)
- (iii) Mutually exclusive events (1 mark)

$$P(A) = 0.3, P(B) = 0.2 \quad P(A \cap B) = 0.1$$

b) If _____ and _____
Determine the following probabilities:

- (i) $P(A')$ (1 marks)

- (ii) $P(A \cap B)$ (2 marks)
- (iii) $P(A' \cup B)$ (3 marks)

c) Let X be a poisson random variable with a probability mass function given as $P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \dots$

find the maximum likelihood estimator of λ (5 marks)

d) Disks of polycarbonate plastics from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

	Shock resistance		
	High	Low	Total
High	70	9	79
Low	16	5	21
Total	86	14	100

(1 mark)

Let A denote the event that a disk has a high shock resistance and let B denote the event that a disk has a high scratch resistance. Calculate:

- (i) $P(A/B)$ (2 marks)
- (ii) $P(B/A)$ (2 marks)
- (iii) Are the events A and B independent (2 marks)
- e) Verify that the following functions are probability mass functions (3 marks)

x	-2	-x	0	1	2
$f(x)$	1/8	2/8	2/8	2/8	1/8

f) The time until a chemical reaction is complete in milliseconds is approximated by the cumulative distribution function.

$$F(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-0.01x} & x \geq 0 \end{cases}$$

Determine:

- (i) The probability density function of X (2 marks)
- (ii) Proportion of reactions which is completed within 200 milliseconds. (2 marks)
- g) Let the continuous random variable X denote the current measured in a thin copper wire in Amperes. $f(x) = 0.05$ for $0 \leq x \leq 20$
- Assume that the range of X is $(0, 20A)$ and assume that the probability density of $f(x) = 0.05$ for $0 \leq x \leq 20$
- . What is the probability that a current measurement is less than 10 milli amperes? (3 marks)

Question Two

- a) Define the following terms:
- (i) A point estimate (2 marks)
 - (ii) Confidence interval (2 marks)
- b) Briefly describe the FOUR properties of a good estimator (8 marks)
- c) Ten measurements of impact energy (J) on specimens of A38 steel cut at 60°C are given as follows: 64.1, 64.7, 64.5, 64.6, 64.5, 64.3, 64.6, 64.8, 64.2, and 64.3. Assume that the impact energy is normally distributed with $\delta = 1J$. Find a 95% confidence interval for J_u , the mean impact energy. (8 marks)

Question Three

- a) The compressive strength of samples of cement can be modeled by a normal distribution with a mean of 6000kg per cm² and standard deviation of 100 kg/cm². What is the probability that:
- (i) A sample's strength is less than 6250. (3 marks)
 - (ii) A sample's strength is between 58000 and 5900 kg/cm²? (4 marks)
 - (iii) What strength is exceeded by 95% of the samples? (4 marks)
- b) In a digital communication channel, assume that the number of bits received in error can be modeled by a binomial random variable, and assume that the probability that a bit is received in error is 1×10^{-5} . If 16 million bits are transmitted, what is the probability that more than 150 errors occur? (5 marks)
- c) Assume that the number of asbestos particles in a squared meter of dust on a surface follows a poisson distribution with a mean of 1,000. If a squared meter of dust is analyzed, what is the probability that less than 950 particles are found? (4 marks)

Question Four

- a) Define the following terms:
- (i) Statistical hypothesis (1 mark)
 - (ii) Type 1 error (1 mark)
- b) Steven Gerald, a wine merchant has collected opinions on grape wine quality from a random sample of his customers. The customers tasted wines made from grapes grown in three regions of country. They rated wine quality on a scale of 1 (best) to 4. The sample data are given below. Do the quality ratings depend on the grape growing region at 5% level of significance? (10 marks)

Quality Rating	1	11	111	Total road
1	15	10	6	31
2	7	13	12	32
3	11	12	8	31
4	3	8	15	26
	36	43	41	120

- c) The boiling points of a sample of 25 mercury metals is given below:

97.8, 97.2, 97.4, 97.6, 97.8, 97.9, 97.9, 98, 98, 98, 98.1, 98.2, 98.3, 98.4, 98.4, 98.4, 98.5, 98.6, 98.7, 98.8, 98.8, 98.9, 98.9 and 99.

$$H_0: \mu = 98.6 \text{ vs } H_a: \mu \neq 98.6$$

Test the hypothesis at 5% level of significance. (8 marks)

Question Five

a) The table below shows two variables, X and Y

X	Y
36	54
26	30
12	28
40	48
24	36
18	30
30	38
30	46
14	16
34	42

- (i) Plot a scatter diagram of the two variables (4 marks)
- (ii) Does Y depend on X (7 marks)
- (iii) Fit a regression model between X and Y (9 marks)