THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE DECREE IN BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING THIRD YEAR FIRST SEMESTER

SMA 2371 : PARTIAL DIFFERENTIAL EQUATIONS

END OF SEMESTER EXAMINATION
SERIES: AUGUST/SEPTEMBER 2011
TIME: 2HOURS

Instructions to Candidates:
Answer Question **ONE** (Compulsory) and any other **TWO** questions
All working must be clearly shown
Maximum marks for each part of a question are as shown
This paper consists of **FOUR** printed pages

QUESTION ONE (30 MARKS)

a Find the orthogonal trajectories of the family of curves $y+1=ax$, where a a constant is and give a geometric description of these trajectories. [5 Marks]

b Obtain the general solution to the partial differential equation

$$(y-x)p+(y+x)q=\frac{x^2+y^2}{z}$$

[5 Marks]

c Derive the partial differential equation arising from

$$z=\frac{1}{2}\left(a^2+2\right)x^2+axy+bx+\phi(y+ax)$$

[6 Marks]

d Show that the sets of parametric equations

$$x = a\sin u\cos v, \quad y = a\sin u\sin v, \quad z = a\cos u$$

and

$$x = a\frac{\left(1-v^2\right)}{1+v^2}\cos u, \quad y = a\frac{\left(1-v^2\right)}{1+v^2}\sin u, \quad x = \frac{2av}{1+v^2}$$

represent the same surface of a sphere, center the origin, O. [6 Marks]

e Find the complete solution of

$$\left(D_x{}^2 - 3D_xD_y + 2D_y^2\right)z = e^{3x+y} - \cos(4x-y)$$

[8 Marks]

QUESTION TWO (20 MARKS)

a Find the general solution of the partial differential equation by direct integration

$$t\frac{\partial^2 u}{\partial x\partial t} + 2\frac{\partial u}{\partial x} = x^2$$

[6 Marks]

b Use the method of separation of variables to solve the one dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

$$u(0,t) = u(L,t) = 0, \quad t \geq 0$$
$$u(x,0) = f(x), \quad 0 \leq x \leq L$$
$$u_t\big|_{t=0} = g(x), \quad 0 \leq x \leq L$$

Satisfying the given Cauchy conditions where f and g are given functions, L is a given constant and $c^2 = \frac{\tau}{\rho}$ [14 Marks]

QUESTION THREE

a Use Laplace transform method to solve the partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to the boundary conditions

$$u(x,0) = 3\sin 2\pi x$$
$$\ell\{u(0,t)\} = U(0,s) = 0$$
$$\ell\{u(L,t)\} = U(L,s) = 0, \quad 0 < x < L, \quad t > 0$$

where $L \in \mathbb{N}$, ($\mathbb{N}$ a natural number) and $\ell\{u(x,t)\}$ denotes the Laplace transform of $u(x,t)$ [10 Marks]

b An infinite metal plate covering the first quadrant has the edge along the y-axis held at 0^0, and the edge along the x-axis held at

$$u(x,0) = \begin{cases} 100^0, & 0 < x < 1 \\ 0^0, & x > 1 \end{cases}$$

Use Fourier transform to find the steady-state temperature distribution as a function of x and y. Assume temperatures of zero as y tends to infinity. [10 Marks]

QUESTION FOUR (20 MARKS)

a Solve the system

$$y_1{}' = 4y_1 - 2y_2$$

$$y_2{}' = y_1 + y_2$$

[12 Marks]

subject to the initial conditions $y_1(0) = 3$ and $y_2(0) = -1$

b Find the General Solution for $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} = 5xy$ [8 Marks]

QUESTION FIVE (20 MARKS)

a Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with the family of planes parallel to $z = 0$. [11 Marks]

b Verify that $z = ax + by + ab$ is the complete solution the partial differential equation $z = px + qy + pq$

Hence show that the integral surface of $z = px + qy + pq$ passing through the curve $x = \tau, \quad y = \tau, \quad z = \tau^2$ can be expressed as

$$z = \frac{\left[x + \left(-3 \pm 2\sqrt{2}\right)y\right]^2}{4\left(3 \mp 2\sqrt{2}\right)}$$

[9 Marks]

A SHORT TABLE OF LAPLACE TRASFORMS

$f(t)$	$L\{f(t)\}$
1	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+1}$
$\frac{\sin at}{t}$	$\tan^{-1}\frac{a}{s} \quad for \operatorname{Re} s > ima$
$\sin at$	$\frac{a}{s^2+a^2}$, $for \operatorname{Re} s > ima$
$\cos at$	$\frac{s}{s^2+a^2}$, $for \operatorname{Re} s > ima$
$\frac{1}{t}\sin at \cos bt$	$\frac{1}{2}\left(\tan^{-1}\frac{a+b}{s}\right) + \tan^{-1}\left(\frac{a-b}{s}\right) for \operatorname{Re} s > 0$