THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING/MECHANICAL ENGINEERING/CIVIL ENGINEERING

AMA 4102/SMA 2107: GEOMETRY

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: FEBRUARY/MARCH 2012
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- Answer booklet

This paper consists of **FIVE** questions
Answer question **ONE (COMPULSORY**) and any other **TWO** questions
This paper consist of **THREE** printed pages

QUESTION ONE (30 MARKS)

a) Prove that

$$\frac{\sin 3A \sin 6A + \sin A \sin 2A}{\sin 3A \cos 6A + \sin A \cos 2A} = \tan 5A$$

(4 marks)

b) Given that $\cos\theta = {}^{5}\!/_{13}$ and $\sin\alpha = {}^{3}\!/_{5}$ such that α is obtuse, work out the possible values of $\tan(\alpha - \theta)$ (4 marks)

c) In triangle PQR, $QR = 3.5$, $RP = 4$ and $PQ = 5$. Calculate the size of angle P and hence find the area of the triangle. (3 marks)

d) Find the equations of the two straight lines which pass through the point $(3,-2)$ and make angles of 60^0 with the line $\sqrt{3}x + y = 1$ (3 marks)

e) Verify that the point $(3,2)$ lies on the circle $x^2 + y^2 - 8x + 2y + 7 = 0$ and find the equation of the tangent at this point. (4 marks)

f) Find the eccentricity, the coordinates of the foci and the equations of the asymptotes of the hyperbola $4x^2 - 9y^2 = 36$ (4 marks)

g) Find the general solution of the equation $2\sin 3x \sin x = 1$ (3 marks)

h) Find the Cartesian equations of

i) $r = a(1 + 2\cos\theta)$ (3 marks)

ii) $r\cos(\theta - \alpha) = P$ (2 marks)

QUESTION TWO (20 MARKS)

a) A triangle ABC has sides a, b, c and semi-perimeter S (so that $a + b + c = 2s$). Show that its area is $\Delta = \sqrt{\{s(s-a)(s-b)(s-c)\}}$ (6 marks)

b) Solve the equation: $\cos 6x + \cos 4x + \cos 2x = 0,$ for values of x from 0^0 to 180^0 inclusive (5 marks)

c) If $\tan\alpha = \frac{1}{5},\ \tan\beta = \frac{4}{19}$ and $\tan\gamma = \frac{2}{5},$ show that $\tan(\alpha + \beta + \gamma) = 1$ (4 marks)

d) The elevations of the top Q of a flagstaff PQ from three distant points A, B, C which are in a horizontal line with P are $\theta,\ 2\theta$ and 3θ respectively. Prove that $AB = 3BC$ approximately. (5 marks)

QUESTION THREE (20 MARKS)

a) Show that the tangents to the ellipse $\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{b^2}\right)=1$ at points whose eccentric angles differ by 90^0 meet on the ellipse $\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{b^2}\right)=2$ (5 marks)

b) The line joining the points $(-1,7)$ and $(23,17)$ is taken as the diameter of a circle. Find the equation of this circle, the length of its radius and the coordinates of its centre. (5 marks)

c) Find

i) The rectangular coordinates of the point whose polar coordinates are $\left(6,\frac{\pi}{3}\right)$ and

ii) The polar coordinates of the point whose Cartesian coordinates are $(-5,12)$ (5 marks)

d) Find x from the equation $2\sin^{-1}x+\sin^{-1}(x^2)=\frac{1}{2}\pi$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Show that $y^2=4a(a-x)$ has a polar equation $r=\frac{2a}{1+\cos\theta}$ (8 marks)

b) Sketch the graph of $r=\frac{1}{\pi}\theta$ where $0\le\theta\le\pi$ (3 marks)

c) A circle passes through the point $A(2,-2)$, $B(3,4)$ and its centre is on the line $x+y=2$. Find its equation. (5 marks)

d) Show that the point of intersection of two perpendicular tangents to a parabola lies on its directrix. (4 marks)

QUESTION FIVE (20 MARKS)

a) If $F(\cos\alpha + \mu\sin\alpha) = \mu W$ where $\mu = \tan\lambda,$ prove that $F = \dfrac{W\sin\lambda}{\cos(\alpha-\lambda)}$ (5 marks)

b) Sho that

$$\sin(A+B+C) = \cos A\cos B\cos C(\tan A + \tan B + \tan C - \tan A\tan B\tan C)$$

(5 marks)

c) A triangle has sides of lengths $m-n,$ m and $m+n$ where $m > n > 0.$ Use the cosine formula to show that if the triangle is obtuse angled, then $\frac{1}{4}m < n < \frac{1}{2}m.$ (5 marks)

d) Solve the triangle in which $B = 60^0,$ $b = 14cm$ and $c = 16cm.$ (5 marks)