



TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN
MECHANICAL ENGINEERING

SMA 2279: LINEAR & BOOLEAN ALGEBRA

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: JULY 2013

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Answer Booklet

This paper consist of **FIVE** questions in **TWO** sections **A & B**

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

$$\vec{a} = (4,0,3) \quad \vec{b} = (1,2,-4)$$

- a) Determine a perpendicular vector to the plane containing \vec{a} and \vec{b} . (3 marks)

$$\sim (p \vee q) \vee (\sim p \wedge q) = \sim p$$

- b) Show that $\sim (p \vee q) \vee (\sim p \wedge q) = \sim p$ (3 marks)

- c) Determine the eigen values for the matrix:

$$A = \begin{pmatrix} -10 & -7 \\ 14 & 11 \end{pmatrix}$$

(4 marks)

d) Find the equation of a line through the point $p(4,-2,5)$ and parallel to the vector $\vec{V} = \vec{i} - 3\vec{j} + \vec{k}$ (3 marks)

e) Find the determinant of the matrix. (4 marks)

$$A = \begin{pmatrix} 1 & -2 & 3 & -1 \\ 1 & 1 & -2 & 0 \\ 2 & 0 & 4 & -5 \\ 1 & 4 & 4 & -6 \end{pmatrix}$$

f) Verify that the proposition $pv \sim (p \wedge q)$ is a tautology. (3 marks)

g) Calculate the area of the triangle ΔPQR given $P = (5,1,-2)$, $Q(4,-4,3)$ and $R(2,4,0)$ (4 marks)

$$\vec{r} = -\vec{i} - \vec{j} + 3\vec{k}$$

h) Normalize the vector (3 marks)

i) Convert $(0.68)^{10}$ to binary correct into 3dp. (3 marks)

SECTION B (Answer any TWO questions from this section)

Question Two

a) Reduce the following matrix A to its row echelon form, determine the rank and identify the basic columns.

$$A = \begin{pmatrix} 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 9 \\ 2 & 6 & 7 & 6 \end{pmatrix}$$

(7 marks)

b) Use the Gaussian elimination method together with back-substitution to solve the following system of equations.

$$4x_2 - 3x_3 = 3$$

$$-x_1 + 7x_2 - 5x_3 = 4$$

$$-x_1 + 8x_2 - 6x_3 = 5$$

(7 marks)

c) Given the two vectors $\vec{r} = (1,-1,2)$ and $\vec{w} = (2,4,1)$. Find:

$$\vec{r} \bullet \vec{w}$$

(i) (2 marks)

(ii) $\vec{r} \times \vec{w}$ (2 marks)

(iii) The angle between \vec{r} and \vec{w} (2 marks)

Question Three

a) Find the truth table for $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$. Hence make a conclusion. (5 marks)

b) Convert the following binary numbers to decimal numbers:

(i) 0.10110 (2 marks)

(ii) 11011 (2 marks)

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$$3x - y - 5z + 8 = 0$$

c) Find the distance from the point Q (4, 1, 2) to the plane P: (3 marks)

$$\begin{vmatrix} 1 & 2 & -1 \\ 3 & 5 & 4 \\ -3 & 1 & 2 \end{vmatrix}$$

d) Find (3 marks)

$$\vec{r} = (v_1, v_2, v_3), \vec{w} = (w_1, w_2, w_3) \quad \theta$$

e) Given that \vec{r} and \vec{w} are non zero vectors and θ is the angle between them. Show that:

$$\cos \theta = \frac{\vec{r} \cdot \vec{w}}{\|\vec{r}\| \|\vec{w}\|}$$

(5 marks)

Question Four

a) Find the matrix X such that $X = AX + B$, where:

$$A = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{pmatrix}$$

and

$$(AB)^{-1} = B^{-1}A^{-1}$$

(9 marks)

b) Show that (3 marks)

c) Using Cramer's rule, solve the system (8 marks)

$$3y + 2x = z + 1$$

$$3x + 2z = 8 - 5y$$

$$3z - 1 = x - 2y$$

Question Five

$$P_1 = -3, 1 - 4 \quad P_2 = (4, 4 - 6)$$

- a) Find the equation of a line through the point P_1 and P_2 in parametric form. Hence find the distance d from the point $P(1, 1, 1)$ to the line. **(7 marks)**

- b) Find the equation of a plane containing the points $Q(1, 0, 3)$, $R(1, 2, 1)$ and $S(6, 1, 6)$ **(6 marks)**

$$\vec{u} \times (\vec{r} \times \vec{w}) \quad \vec{u} = (1, 1, 1), \quad \vec{r} = (3, 0, 2) \quad \vec{w} = (2, 2, 2)$$

- c) Calculate $\vec{u} \times (\vec{r} \times \vec{w})$ given that $\vec{u} = (1, 1, 1)$ and $\vec{w} = (2, 2, 2)$ **(4 marks)**
- d) Let P be “Erick reads Nation” q be “Erick reads standard” and r be “Erick reads the star”. Write the following in symbolic form.
- (i) Erick reads Nation or Standard but not the star. **(1 mark)**
 - (ii) Erick reads Nation and standard, or he does not read Nation and The star. **(1 mark)**
 - (iii) It is not true that Erick reads the Star or Standard but not Nation **(1 mark)**