



TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN
CIVIL ENG./MECHANICAL ENG/ELECTRICAL & ELECTRONIC ENG.

SMA 2270: CALCULUS III

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: JULY 2013

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Answer Booklet

This paper consist of **FIVE** questions in **TWO** sections **A & B**

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One (30 marks)

- a) Given that $w = f(x, y)$ where $x = r \cos \theta$ and $y = r \sin \theta$. Show that

$$\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial w}{\partial \theta}\right)^2 = f_x^2 + f_y^2$$

(6 marks)

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{1+4n^2}}$$

b) Determine the convergence of the series using the limit comparison test. (4 marks)

c) Evaluate each of the following limits:

$$\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12} - 12}{x - 2}$$

i) (4 marks)

$$\lim_{x \rightarrow \pi} \left(\frac{\tan^2 x}{1 - \sec x} \right)$$

ii) (3 marks)

$$\int_0^3 \frac{dx}{(x-1)^{2/3}}$$

d) Evaluate the improper integral (5 marks)

$$\int_1^2 \int_2^3 e^{x+y} dy dx$$

e) Evaluate (3 marks)

$$\ln(1 + e^x)$$

f) Expand in ascending powers of x up to the term containing x⁴. (5 marks)

SECTION B (Answer any TWO questions from this section)

Question Two (20 marks)

$$f(x) = (x-1)(x-2)(x-3)$$

a) Verify the mean value theorem for the function in [0,4] (6 marks)

$$\lim_{x \rightarrow 0} x \ln \tan x$$

b) Evaluate (5 marks)

$$f(x) = |x|$$

c) Investigate whether is differentiable at x = 0. (4 marks)

$$a_n = \sqrt{n^2 + 4n} - n$$

d) Determine the convergence or divergence of the sequence (5 marks)

Question Three (20 marks)

$$f(x) = e^x \sin x \quad [0, \pi]$$

a) Verify Rolle's theorem of in (5 marks)

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

b) Use integral test to determine the convergence or divergence of (4 marks)

$$\theta = -\frac{\pi}{4} \quad \theta = \frac{\pi}{4}$$

- c) Find the area of the region bounded by the rays $\theta = -\frac{\pi}{4}$ and $\theta = \frac{\pi}{4}$ and the graph of the equation $r = 1 + \sin \theta$ (6 marks)

- d) Investigate the continuity of the function below at $x = 4$.

$$f(x) = \begin{cases} \frac{1}{x-3} & x \geq 4 \\ 5-x & x < 4 \end{cases}$$

(5 marks)

Question Four (20 marks)

- a) Expand $\ln x$ about $x = 1$ up to the fourth degree term and hence obtain $\ln(1.1)$ (6 marks)

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

- b) Find (8 marks)

$$-\frac{2}{1}, \frac{8}{2}, -\frac{26}{6}, \frac{80}{24}, -\frac{242}{120}, \dots$$

- c) Find the n th term of the sequence and determine whether the sequence converges or diverges. (6 marks)

Question Five (20 marks)

- a) Evaluate $\iint_R xy(x+y) dx dy$ over the region between $y = x^2$ and $y = x$ (6 marks)

- b) Find the total derivative given that $z = f(x, y) = x^2 + 2xy + 4y^2$ and $y = e^{ax}$ (4 marks)

$$f(x, y) = \frac{2y}{y + \cos x} \quad f_x \quad f_y$$

- c) Given that , find and . (4 marks)

- d) Find the interval of convergence of the series.

$$\sum_{n=0}^{\infty} \frac{(x+3)^n}{(n+1)4^n}$$

(6 marks)