



THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)

(A Centre of Excellence)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE IN BACHELOR OF SC./ENG. IN
ELECTRICAL & ELECTRONICS/ MECHANICAL & AUTOMOTIVE &
BUILDING & CIVIL ENGINEERING

SMA 2172/AMA 4102: CALCULUS I

END OF SEMESTER EXAMINATION

SERIES: AUGUST 2012

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Answer Booklet

This paper consist of **FIVE** questions in **TWO** sections **A & B**

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One (30 marks)

a) Define the following terms as used in Algebra:

i) A function

(2 marks)

ii) Local points

(2 marks)

$$y = \frac{1}{36 - x^2}$$

b) Calculate the domain and range of function

(4 marks)

$$f(x) = 3x^2, g(x) = \frac{1}{\sqrt{1+x}}, h(x) = \frac{1+x}{x}$$

$$L = hgf$$

- c) Given that $f(x) = 3x^2$ and $h(x) = \frac{1+x}{x}$ find the composite function $L = hgf$ (3 marks)

$$\frac{x^3 - 125}{x - 5} \quad x \rightarrow 5$$

- d) Find the limit of the function $\frac{x^3 - 125}{x - 5}$ as $x \rightarrow 5$ (4 marks)

$$f(t) = kt^4$$

- e) Find from the first principles, the derivative of the function $f(t) = kt^4$ (4 marks)

$$x = 80t \quad y = 64t - 16t^2, \quad \frac{dy}{dx} = 0$$

- f) Given that $x = 80t$ and $y = 64t - 16t^2$, find the value of t for which $\frac{dy}{dx} = 0$ (4 marks)

$$y = x \sec^2 3x - \ln \tan x$$

- g) Show that the derivative of the function $y = x \sec^2 3x - \ln \tan x$ is $\sec^2 3x - 6x \sin 3x \cos 3x - \cot x + \tan x$ (4 marks)

$$g(x) = 5 + \frac{x}{2} \quad g^{-1}(x)$$

- h) Given that the function $g(x) = 5 + \frac{x}{2}$ find the value of $g^{-1}(x)$ (3 marks)

SECTION B (Answer any TWO questions from this section)

Question Two (20 marks)

$$\frac{dy}{dx} x^5 + 4xy^3 - 3y^5 = 2$$

- a) Find $\frac{dy}{dx}$ if $x^5 + 4xy^3 - 3y^5 = 2$ (4 marks)

$$\frac{dy}{dx} y^{2/3} = \frac{(x^2 + 1)(3x + 4)^{1/2}}{\sqrt{(2x - 3)(x^2 - 4)}}$$

- b) By using logarithmic differentiation find $\frac{dy}{dx}$ if $y = 3 \ln \sin x$ (6 marks)

- c) Calculate the derivative of $y = 3 \ln \sin x$ (4 marks)

- d) One side of a rectangle is three times the other. If the perimeter is increased by 2% what is the percentage change in the area? (6 marks)

Question Three (20 marks)

- a) 1000m of fencing wire is to be used to make a rectangular enclosure. Find the greatest possible area and the corresponding dimensions. (3 marks)

$$y = 2x^3 + 3x^2 - 12x + 7$$

- b) Find the turning points of the graph $y = 2x^3 + 3x^2 - 12x + 7$ (4 marks)

- i) Distinguish between maximum and minimum values of the points obtained above (4 marks)

- ii) Show that the graph passes through (1,0) and find the other point on the x axis. (2 marks)

iii) Sketch the above curve. (2 marks)

Question Four (20 marks)

a) Find the equations of the normal to the parabola $4y = x^2$ at the points $(-2,1)$ and $(-4,4)$ (5 marks)

b) The curve $y = (x - 2)(x - 3)(x - 4)$ cuts the x-axis at the point P (2, 0) Q (3, 0) and R (4, 0). Prove that the tangents at P and R are parallel. (5 marks)

c) At what point does the normal to the curve at (q, y) cut the y axis? (5 marks)

Question Five (20 marks)

a) The side of a cube is increasing at the rate of 6cm/s. Find the rate of increase of the volume when the length of a side is 9cm. (9 marks)

b) By applying the concepts of small changes as used in calculus, find the approximate value of $\sqrt{627}$ (5 marks)

c) A particle move along a straight line so that aft ts, its distance from O a fixed point on the line is 5m where $s = t^3 - 3t^2 + 2t$. Calculate.

- i) The time when the particle is at O (4 marks)
- ii) The velocity and acceleration at the times calculated in part (i) above. (3 marks)
- iii) What is its average velocity during the first second? (2 marks)