TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE

SMA 2100: DISCRETE MATHEMATICS

END OF SEMESTER EXAMINATION
SERIES: APRIL 2013
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) Using suitable examples, explain each of the following:

(i) A tantology **(2 marks)**
(ii) A contingency **(2 marks)**
(iii) A contradiction **(2 marks)**

b) Let: $A\{P,3,k,\{t\}\}$ and $B=\{x,t,m\}$

State with reasons whether each of the following is true or false.

(i) $\{x\} \in B$ **(2 marks)**

(ii) $t \in A$ **(2 marks)**

(iii) $\{x\} \subset B$ **(2 marks)**

(iv) $\phi \subset A$ **(2 marks)**

c) Suppose $f : IR \to IR,\ f(x) = 2^{x+1}$

(i) Check whether f is injective **(2 marks)**

(ii) Check whether f is surjective **(2 marks)**

d) Construct the truth table of $[\sim (p \wedge q)] \vee r$ **(4 marks)**

e) Use mathematical induction to show that $n^3 - n$ is divisible by 3 **(6 marks)**

f) If $f, g : IR \to IR$ and $f(x) = 2x^2 + 1, g(x) = 7x + 6$.

Find:

$fog(x)$

(2 marks)

SECTION B (Answer any TWO questions from this section)

Question Two

a) A survey conducted in TUM revealed that in a class of 80 students, 55 prefer google search engine, 46 prefer yahoo search engine while 50 prefer MSN. Also 37 prefer google and yahoo while 28 frefer google and MSN. 7 students do not prefer any of the three and 12 prefer all the three. By use of a venn diagram, find the number of students who prefer exactly one of the search engine. (8 marks)

b) Let:

$A = \{a, b, c, d, 1, 2, 3, k, t, f\}$

$B = \{1,2,3,4,5\},\ C = \{a, b, c, d, e, f\}$

If $U = \{a, b, c, \text{-----}, t, 1, 2, 3, 4, 5\}$ find:

(i) $A^C \cap B$

(ii) $A^C \cup (A \cap B^C) \cup B$

(iii) $(A - B) \cap (B - C)$

(iv) Show that $(A - B) \cap (B - A) = \phi$ **(8 marks)**

c) Prove the following using set notation. **(4 marks)**

$(A \cap B)^C = A^C \cup B^C$

(4 marks)

Question Three

a) Let

$f : IR \to IR,\ f(x) = 3x^2 + 1$

$g : IR \to IR, g(x) = \dfrac{x^3 - 5x}{7 - 4x}$

(i) Compute fog and gof stating the domain **(4 marks)**

(ii) Compute $g^{-1}(2)$ **(3 marks)**

b) Use direct proof to show that if q is even then 4 divides q^2. **(4 marks)**

c) Use mathematical induction to show that $3 + 6 + 9 + \_\_\_ 3n = \frac{3}{2} n(n+1)$ **(5 marks)**

d) Show that if $f : X \to Y$ is a bijection then $|X| = |Y|$ **(4 marks)**

Question Four

a) (i) Explain what is meant by logical equivalence. **(3 marks)**

(ii) Let p and q be proposition. Show that:

$(p \vee q) \wedge (p \vee \sim q) \cap (\sim p \vee q) \Leftrightarrow p \wedge q$

(5 marks)

b) Write the inverse and the contrapositive of the statement: "If I come early then I can get the car" **(4 marks)**

c) Test the validity of the following argument:
"If Jane becomes the president, productivity will increase" Productivity decreased therefore Jane did not become the president. **(4 marks)**

d) Negate the following statements:

(i) If Mark appreciates Discrete Mathematics, then he will become a pure Mathematics major **(2 marks)**

(ii) $\forall x \in IR,\ x^2 > 0$ **(2 marks)**

Question Five

a) Use contradiction to prove that $\sqrt{2}$ is irrational. **(6 marks)**

b) With the negation of each of the following proportions:

(i) $Vx \in IR,\ x > 3 \rightarrow x^2 > 9$ **(2 marks)**

(ii) Every polynomial function is continuous. **(2 marks)**

(iii) There exists a triangle with the property that the sum of angles is greater than 180o **(2 marks)**

c) Consider the universal conditional proposition:

$Vx \in D,$ if $P(x)$ then $Q(x)$

(i) Find the contra positive **(2 marks)**

(ii) Find the converse **(2 marks)**

d) Write in the form $Vx \in D, P(x)$ the proposition "every real number is either positive, negative or 0" **(2 marks)**

e) If C is contradiction, determine the validity of the following for any p. **(2 marks)**

$\sim P \rightarrow C$

$\therefore P$