TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF MATHEMATICS & COMPUTER SCIENCE

AMA 4314: REAL ANALYSIS I

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Show that $\forall a \in \Re, -a$ is unique under addition **(4 marks)**

b) Show that if $x \neq 0,$ then $x^2 > 0$ and hence show that $1>0$ **(4 marks)**

c) Show that $\forall x, y \in \Re$ $x \cdot (-y) = -(xy)$ **(2 marks)**

d) Use the axioms of numbers to show that $0.x = 0$ $\forall x \in \Re$ **(3 marks)**

e) Determine whether $S=\{\frac{n}{n+1}:n\in \mathbb{N}\}$ is closed or open. **(3 marks)**

f) Show that every subset of a countable set is countable **(4 marks)**

g) Use Cauchy's root test to determine the convergence of:

$$\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^{n}$$

(3 marks)

h) Find the sum of the series: $\sum_{n=1}^{\infty}\frac{1}{3^{n+1}}$ **(3 marks)**

i) Obtain the derived sets of the following:

(i) $\{x\in \Re:0<x<1\}$ **(2 marks)**

(ii) $\{1,-1,1\tfrac{1}{2}-1\tfrac{1}{2},-1\tfrac{1}{3}\ldots\}$ **(1 mark)**

j) Test whether $A=\{x\in \Re:x>1\}$ is an inductive set **(1 mark)**

Question Two

a) Show that every convergent sequence is a Cauchy sequence **(6 marks)**

b) Show that the union of any finite number of closed sets is closed **(6 marks)**

c) Prove that every convergent sequence is bounded **(4 marks)**

d) State and prove the Bolzano-Weirstrass theorem **(4 marks)**

Question Three

a) Show that the function f(x) = 5x is uniformly continuous on $\Re$ **(5 marks)**

b) In the set of real numbers define the function d: $\Re\times\Re\rightarrow\Re$ by d(x, y) = |x – y| where | . | donates the absolute value function. Show that d is a metric and $(\Re,d)$ is a metric space **(6 marks)**

c) Use D'Alembert's test to set for the convergence of:

$$\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$$

(4 marks)

d) Show that a sequence of real numbers converges to a real number if and only if all its subsequences converges to the same real number **(5 marks)**

Question Four

a) Let $f(x)=1+\sin x$ for $0\le x\le 2\pi$ Test whether f is Riemann integrable given partitions as $\left\{0,\ {2\pi}/{3},\pi,{4\pi}/{3},\ 2\pi\right\}$

(7 marks)

b) Show that $f(x)=x$ is Riemann integrable on [0, 1] and hence find $\int_0^1 x\,dx = {1}/{2}$ **(8 marks)**

c) Show that Riemann integral is linear **(5 marks)**

Question Five

a) State and prove the sandwich theorem. **(5 marks)**

b) Show that $f(x)=x^2+2x+6$ is continuous at x = 3 **(5 marks)**

c) Show that a set F is closed if F= $\overline{F}$ **(5 marks)**

d) Use Leibnitz test to test the convergence of:

$$\sum_{n=1}^{\infty}\frac{1}{n}(-1)^n$$

(5 marks)