TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE
BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY (BSIT 12S)

SMA 2230: PROBABILITY & STATISTICS II

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: MARCH 2014
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **FOUR** printed pages

SECTION A (COMPULSORY)

Question One

a) Define probability of an event **(2 marks)**

b) Let X be a random variable with probability density function:

$$f(x)=\begin{cases}\frac{1}{5}x+k & 0\le x\le 3\\ 0 & otherwise\end{cases}$$

Find:
(i) The value of k **(3 marks)**

$P(1 \leq x \leq 3)$

(ii) **(3 marks)**

c) Each sample of water has a 15% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 20 samples, exactly 3 contain the pollutant. Give your answer correct to 4 decimal places. **(3 marks)**

d) The mean weight of 200 college students is 72 kg with a standard deviation 6 kg. Assuming the weights are normally distributed, find the probability that students weight:

(i) More than 82 kg

(ii) Between 60 and 73 kg and hence the number of students with weight between 60 and 73 kg. **(3 marks)**

e) A shipment of 20 digital voice recorders contains 5 that are defective. If 10 of them are randomly chosen for inspection, determine the probability that:

(i) 3 of the ten will be defective **(3 marks)**

(ii) Less than 3 of the ten will be defective. **(4 marks)**

f) As part of a quality improvement project at TUM kitchen focusing on the number of students served per minute during lunch time, data were gathered on a one day as shown in Table 1.

No of students served in one minute, x	P(x)
0	0.05
1	0.20
2	0.45
3	0.20
4	0.10

Table 1

Determine: (i) the mean **(3 marks)**

(ii) variance **(4 marks)**

SECTION B (Answer any TWO questions from this section)

Question Two

a) Define a probability mass function **(3 marks)**

b) An IT equipment franchise holder intends to introduce a new computer model into the market. The company estimates that the model will be very successful with probability of 0.5, moderately successful with a probability of 0.3 and not successful with probability 0.2. The estimated yearly profit associated with the model being very successful is kshs 20 million, being moderately successful is ksh 5 million, not successful results in a loss of kshs 1 million.

Let X be the yearly profit (in millions, kshs? New model. Determine the probability mass function of X **(3 marks)**

c) A digital communication system operates by sending and receiving bits. Due to uncontrolled factors, some bits are received in error. Let X denote the number of bits received in error. If the probability of a bit being received in error p = 0.001, determine when 10 bits are transmitted, the probability that:

(i) Only one bit is received in error **(2 marks)**

(ii) At least one bit is received in error **(3 marks)**

(iii) At most two bits are received in error **(4 marks)**

(iv)

d) A section of a high-way is known to experience an average 3 accidents per week. Determine the probability that:

(i) No accident occurs at the section in period of one week. **(2 marks)**

(ii) Less than 3 accidents occur at the section in a period of 4 weeks. **(3 marks)**

Question Three

a) A firm manufactures disk drive bearings whose diameters are normally distributed with parameters $\mu = 1,\ \sigma = 0.002$ in centimeters. The buyer's specifications require that these diameters be 1.000 1.000 ± 0.003.

(i) Determine the fraction of the manufacturer's bearings that are likely to be rejected. **(6 marks)**

(ii) If the manufacturer decides to improve the quality control by reducing σ, determine the value of σ that ensures that no more than 2% of the bearings are rejected. **(4 marks)**

b) The loaves of bread distributed by Salim's bakery in Mombasa have an average length of 30cm and a standard deviation of 2cm. A sample of 25 loaves of bread is selected from a shift's collection. Assuming normal distribution of bread length, determine:

(i) The number of loaves likely to have a length more than 30.7cm. **(5 marks)**

(ii) The number of loaves likely to have length between 29.3 and 30.7cm. **(3 marks)**

(iii) The number of loaves to be recycled.

Question Four

a) Research by the Ministry Roads in Kenya shows that the proportion of highway sections requiring repairs in a given year is a random variable having a beta distribution with $\alpha = 3$ and $\beta = 2$.

(i) Determine f(x) **(3 marks)**

(ii) Sketch the graph of f(x) **(4 marks)**

(iii) Find the average per centage of highway sections requiring repairs in any given year. **(1 mark)**

(iv) Find the probability that at most half of the highway sections will require repairs in any given year. **(2 marks)**

b) It is known from previous data that the length of time in months between customers' complaints is a gamma distribution with $\alpha = 2$ and $\beta = 4$. Changes were made that involved tightening of quality control requirements. Following these changes, it took 20 months before the first complaint. Determine whether the quality control tightening was effective. **(10 marks)**

Question Five

a) Define a moment generating of a random variable X.

b) Let X be a continuous random variable having p.d.f:

$$f(x) = \begin{cases} \frac{1}{3} & - < x < 2 \\ 0 & elsewhere \end{cases}$$

Show that its moment generating function is given by:

$$M(t) = \begin{cases} \frac{e^{2t} - e^{-t}}{3t} & t \neq 0 \\ 1 & t = 0 \end{cases}$$

c) A consumer products company is formulating a new shampoo and is interested in foam height (mm). Foam height is normally distributed with standard deviation of 20mm. The company wishes to test the hypothesis:

$H_o : \mu = 175$

against the alternative

$H_1 : \mu < 175$

Using n = 10, $\overline{X} = 190$ MM

(i) What conclusions would you reach? **(6 marks)**

(ii) What's the probability that you would observe a sample as large as 190mm or larger, if the true mean foam height was 175mm **(2 marks)**

(iii) State the type I error and your reasons **(2 marks)**