TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY (BSIT)

SMA 2230: PROBABILITY & STATISTICS II

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) (i) Give TWO properties of a probability density function. **(2 marks)**
(ii) Define a random variable **(1 m ark)**

b) Let C be a constant and consider the density function for the random variable Y:

$$f(g) = \begin{cases} y^2, & 0 \le y \le 2 \\ 0, & elsewhere \end{cases}$$

(i) Find the value of C **(3 marks)**
(ii) Find the cumulative distribution function f(y) **(3 marks)**
(iii) Find E(Y) **(3 marks)**
(iv) Find Var (Y) **(3 marks)**

c) In a semiconductor manufacturing process. Three wafers from a lot are tested. Each wafer is classified as pass or fail. Assume that the probability that a wafer passes the test is 0.8 and that wafers are independent. Determine the probability mass function of the number of wafers from a lot that pass the test. **(5 marks)**

d) If X is binomially distributed with 6 trials and probability of success is equal to ¼ at each attempt, what is the probability of;

- **(i)** Exactly 4 successes **(2 marks)**
- **(ii)** At least one success **(2 marks)**

e) From a long term experience a factory owner knows that a worker can produce a product in an average time of 89 minutes. However, on Monday morning there is the impression that it takes longer. To test whether this impression is correct a sample (n=12) is taken with mean of 92.2 and standard deviation of 10.75. We assume that the production time is normal. Verify whether this impression is correct at 5% significance level:

- **(i)** State the null and alternative hypothesis **(2 marks)**
- **(ii)** Compute the test statistics at 5% significance level **(2 marks)**
- **(iii)** What are your conclusions **(2 marks)**

Question Two

a) A telephone operator handles on average 5 calls every 3 minutes:

- (i) What is the probability that there will be no calls in the next minute?
- (ii) What is the probability that there will be at least two calls?

(Hint: Use Poisson Distribution) **(6 marks)**

b) The probability that a light bulb will fail on any given day is 0.001, what is the probability that it will last at least 30 days. (Hint: Use Geometric Distribution) **(4 marks)**

c) Benzene is a possible cancer-agent. It is suspected that the concentration of Benzene in the air from a chemical company is greater than 1ppm. The following sample is collected to test this claim:

0.21	1.44	2.54	2.97	0.00
3.91	2.24	2.41	4.50	0.15
0.30	0.36	4.50	5.03	0.00
2.89	4.71	0.85	2.60	1.26

- (i) State the null and alternative hypothesis **(2 marks)**
- (ii) Test the hypothesis **(5 marks)**
- (iii) What is your conclusion **(2 marks)**

Question Three

a) Batches that consist of 50 coil springs from a production process are checked for conformance to customer requirements; the mean number of nonconforming coil springs in a batch is 5. Assume that the number of nonconforming springs in a batch denoted as X, is a binomial random variable:

- (i) What are n and p? **(3 marks)**
- (ii) What is the probability that the number of nonconforming springs is at most 2 **(4 marks)**

(iii) Find $P(X \geq 49)$ **(3 marks)**

b) The number of flaws in bolts of cloth in textile manufacturing is assumed to be Poisson distributed with a mean of 0.1 flaw per square meters.
   (i) What is the probability that there are two flaws in a 1 square meter of cloth? **(3 marks)**
   (ii) What is the probability that there is one flaw in 10 square meters of cloth? **(3 marks)**
   (iii) What is the probability that there are no flaws in 20 square meters of cloth? **(3 marks)**

c) Write the formula of the variance of a random variable following binomial distribution. **(1 mark)**

Question Four

a) Assume X is normally distributed with a mean of 10 and a standard deviation of 2. Determine the following:
   (i) $P(X < 13)$ **(2 marks)**
   (ii) $P(6 < X < 14)$ **(3 marks)**
   (iii) $P(X > 9)$ **(3 marks)**

b) $$f(x) = \begin{cases} x/8 & \text{if } 3 < x < 5 \\ 0 & \text{if elsewhere} \end{cases}$$

Determine the following probabilities:

   (i) $P(X > 3.5)$ **(3 marks)**
   (ii) $P(4 < X < 5)$ **(3 marks)**
   (iii) $P(X < 4)$ **(2 marks)**
   (iv) Determine the mean of x **(2 marks)**
   (v) Determine the variance of x **(2 marks)**

Question Five

a) Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The fill volume can be assumed normal with standard deviation $\delta_1 = 0.02$ and $\delta_2 = 0.025$ ounces. A member of the Quality Engineering staff suspects that both machines fill to the same mean net volume, whether or not this volume is 16.0 ounces. A random sample of 10 bottles is taken from the output of each machine.

Machine 1		**Machine 2**	
16.03	16.01	16.02	16.03
16.04	15.96	15.97	16.04
16.05	15.98	15.96	16.02
16.05	16.02	16.01	16.01
16.02	15.99	15.99	16.00

Do you think the Engineer is correct 2 use $\alpha = 0.05$ **(12 marks)**

b) An Engineer who is studying the tensile strength of a steel alloy intended for use in golf club shafts knows that tensile strength is approximately normally distributed with $\delta = 60\, ps$: A random sample of 12 specimens has a mean tensile strength of $\bar{x} = 3250$ ps. Test the hypothesis that mean strength is 3500 PS: Use $\alpha = 0.01$ **(8 marks)**