TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN INFORMATION COMMUNICATION TECHNOLOGY

SMA 2230: PROBABILITY & STATISTICS II

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) (i) Give TWO properties of a probability mass function **(2 marks)**

(ii) Define a random variable **(1 mark)**

b) Let K be a constant and consider the density function for the random variable X:

$$fx = \begin{pmatrix} kx^2, & 0 \le x \le \\ 0 & elsewhere \end{pmatrix}$$

Define the following terms:

(i) Find the value of K **(3 marks)**

$P(1 \le 1x \le 1.5)$

(ii) Find **(3 marks)**

(iii) Find F(1) **(3 marks)**

(iv) Find E(x) and Var (x) **(6 marks)**

c) An assembly consists of THREE mechanical components. Suppose that the probabilities that the first, second and third components meet specifications are 0.95, 0.98 and 0.99. Assume that the components are independent.

(i) Determine the probability mass function of the number of components in the assembly that meet specification **(4 marks)**

(ii) Plot the probability mass function in a graph **(2 marks)**

d) If X is binomially distributed with 5 trials and probability of success is equal to ¼ at each attempt, what is the probability of:

(i) Exactly 3 success **(2 marks)**

(ii) At least 2 success **(2 marks)**

e) The variance tells us the center of a distribution:

(i) True or false **(1 mark)**

(ii) Explain your answer **(1 mark)**

Question Two

a) Environmental Science and Technology (Oct.1993) reported on a study of contaminated soil in the Netherlands. A total of 72 soil specimens were sampled, dried and analyzed for the contaminant cyanide. The cyclical concentration (milligrams per kilogram of soil) of each soil specimen was determined using an infrared microscope method. The sample resulted in a mean cyanicle level of $\bar{x}$ = 84mg/kg.

(i) State the hypothesis that the true mean cycinicle level in soil in Netherlands falls below 100mg/kg. Use $\alpha = 10\%$ **(2 marks)**

(ii) Compute the test statistics at 10% level of significance **(2 marks)**

(iii) What is your conclusion **(3 marks)**

(iv) Would you reach the same conclusion in (iii) above using $\alpha = 0.05\%$ **(3 marks)**

b) A dust mite allergen level that exceeds 2 micrograms per gram (ug/g) of dust has been associated with the development of allergies. Consider a random sample of four homes and let Y be the number of homes with a dust mite level that exceeds 2 μg/g. The probability distribution of Y, based on a May 2000 study by the National Institute of Environmental Health sciences, is as shown in the following table:

y	0	1	2	3	4
p(y)	0.09	0.30	0.37	0.20	0.04

(i) Verify that p(y) is a probability mass function **(1 mark)**

(ii) Find the probability that three or four of the homes in the sample have a dust mite level that exceeds $2\mu g / g$ **(3 marks)**

(iii) Find E(y) **(3 marks)**

(iv) Find standard deviation **(3 marks)**

Question Three

a) The phone lines to a airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline.

(i) What is the probability that for exactly three calls the lines are occupied **(3 marks)**

(ii) What is the probability that for at least one call the lines are not occupied **(3 marks)**

(iii) What is the expected number of calls in which the lines are all occupied **(3 marks)**

b) A lot of 35 washers contains 5 in which variability in thickness around the circumference of the washer is unacceptable. A sample of 10 washers is selected at random without replacement.

(i) What is the probability that none of the unacceptable washers is in the sample **(3 marks)**

(ii) What is the probability that at least one acceptable washer is in the sample **(3 marks)**

(iii) What is the probability that exactly one unacceptable washer is in the sample **(3 marks)**

c) Two fair dice are rolled. What is the probability that the sum of the outcomes equal exactly 7 **(2 marks)**

Question Four

a) The line width of semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.

(i) What is the probability that a line width is greater than 0.62 micrometer **(3 marks)**

(ii) What is the probability that a line width is between 0.47 and 0.63 micrometer **(3 marks)**

(iii) The line width of 90% of samples is below what value? **(4 marks)**

b) Let $f(x) = (K3x^2 + 1) \quad for \; 0 \le x \le 2$. Find:

(i) The value of K that makes the given function of PDF on the interval **(2 marks)**

(ii) The mean and variance of the random variable X **(5 marks)**

c) If X is normally distributed with mean of 10 and a standard deviation of 2. Determine the $P(x < 13)$ **(3 marks)**

Question Five

a) The number of episodes per year of otitis media, a common disease of the middle ear in early childhood, follows a Poisson distribution with parameter $\lambda = 1.6$ episodes per year. An interesting question in pediatrics is whether the tendency for children to have many episodes of otitis media is inherited in a family:

(i) Find the probability of getting 3 or more episodes of otitis media in the first 2 years of life **(4 marks)**

(ii) Find the probability of not getting any episodes of otitis media in the first year of life **(3 marks)**

$$f(x)=\begin{cases} x/8 & \text{if } 3<x<5 \\ 0 & \text{if elsewhere} \end{cases}$$

b)

Determine the following probabilities:

(i) $P(x>4.5)$ **(3 marks)**

(ii) $P(4<x<5)$ **(3 marks)**

(iii) $P(x<4)$ **(3 marks)**

(iv) Determine the mean and variance of x **(4 marks)**

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