TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF TECHNOLOGY IN APPLIED TECHNOLOGY (BTAP)

AMA 4212: VECTOR ANALYSIS

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **TWO** printed pages

Question One (Compulsory)

a) Find the angle θ between the vector $v=(2,1,-1)$ and $w=(3,-4,1)$ **(4 marks)**

b) Prove that the area of a parallelogram PQRS with sides v and w is $\| vxw \|$ **(4 marks)**

c) Consider vectors $a=i+3j-k,\ b=-i+j-5k$ and $c=3i-2j+7k$. Find the vector $(a\times b\times c)$ **(4 marks)**

d) Define the following terms:

(i) Divergence of vector v(x, y, z) **(2 marks)**

(ii) Cure of vector $v(x,y,z)$ **(2 marks)**

e) Find the unit tangent vector at any point on the curve $x = t + 2t^2, = 3t - 4, \; z = -6t + 2$ when t = 2 **(5 marks)**

f) Let C be the curve $x = 1 - y^2$ from (0, -1) to (0, 1). Evaluate:

$$\int_C y^3 dx + c^2 dy$$

(5 marks)

g) Convert the point (-2, -2, 1) from Cartesian coordinate to:

(i) Cylindrical coordinates

(ii) Spherical coordinates **(4 marks)**

Question Two

a) State Green's theorem in the plane **(4 marks)**

b) Verify Green's theorem in the plane **(8 marks)**

$$\oint 3x^2 - 8y^2 dx + 4y - 6xydy$$

where C is the boundary of the region defined by $y = \sqrt{x}, = x^2$

c) Show that the vectors $A = 3i - 2j + k, \; B = i - 3j + 5k, \; C = 2i + j - 4k$ form a right triangle. Support your answer **(8 marks)**

Question Three

a) Find the divergence and curl of the vector $v = xyzi + 3x^2yj + \left(xz^2 - y2z\right)k$ at the pt (3, 1, -2) **(8 marks)**

b) Calculate the area of the triangle PQR where P(2, 4, -7), Q = (3, 7, 18) and R = (-5, 12, 8) **(8 marks)**

c) Find $grad\phi$ if $\phi = x^3 + y^3 + 3xyz$ **(4 marks)**

Question Four

a) The acceleration of a particle at any time t is given by $\vec{a} = 18\cos 3t \; i - 3\sin 2tj + 6tR$. If the velocity $\vec{v}$ and displacement $\vec{r}$ are zero at t = 0, find $\vec{v}$ and $\vec{r}$ at any time t. **(10 marks)**

b) State Stoke's theorem relating the lien and the surface integrals **(4 marks)**

c) Evaluate $\oint_C F.dr$ by Stoke's theorem, where $F = y^2 i + x^2 j - \left(x + z\right)k$ and C is the boundary of the triangle with vertices at (0, 0, 0), ((1, 0, 0) and (1, 3, 0) **(6 marks)**

Question Five

$$f(t)=\frac{\cos t}{1+a^2t^2},\frac{\sin t}{\sqrt{1+a^2t^2}},\ \frac{-at}{\sqrt{1+a^2t^2}},\ with\ a\neq 0$$

a) Let $f(t)$ as above, show that $\| f(t) \| = 1$ for all t **(5 marks)**

b) Write the equation $(x-2)^2+(y-1)2+z^2=9$ in spherical coordinate **(5 marks)**

c) Find $Ux(v\times w)$ for $U=(1,2,4), v=2,2,0,\ w=(1,3,0)$ **(5 marks)**

d) (i) Define the gradient function f(x, y) in $\Re^2$ **(2 marks)**

(ii) Determine $\frac{\partial f}{\partial x}(x,y)$ and $\frac{\partial f}{\partial y}(x,y)$ for the function $f(x,y)=x^2y+y+y^3$ **(2 marks)**

(iii) Find gradient f(x, y) at point (1, 2) **(1 mark)**