# TECHNICAL UNIVERSITY OF MOMBASA

# Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

**BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING**
**BACHELOR OF TECHNOLOGY IN RENEWABLE ENERGY**
**BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS**
**(BSME/BTRE/BTAP)**

AAMA 4209: CALCULUS III

**SPECIAL/SUPPLEMENTARY EXAMINATION**
SERIES: JUNE 2015
**TIME ALLOWED:** 2 HOURS

**<u>Instructions to Candidates:</u>**
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

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**Question One (Compulsory)**

**a)** Use ratio test to determine the convergence or divergence of $\sum_{n=0}^{\infty} \frac{2n}{n!}$ **(3 marks)**

**b)** It took 20 seconds for a thermometer to rise from 10ºF to 212ºF where it was taken from a freezer and placed in boiling water. Show that somewhere along the way the mercury was rising at exactly 10.1ºF/s **(3 marks)**

**c)** Express as a ratio of two integers the sum to infinity the recurring decimal $6.2\dot{5}\dot{4}5\dot{4}$ **(5 marks)**

**d)** Evaluate the improper integral below $\int_0^4 \frac{dx}{\sqrt[3]{x-1}}$ **(5 marks)**

**e)** Find the area of the region lying inside the circle $r = 3\cos\theta$ on the positive x-axis and outside the candiod $r = 1 + \cos\theta$ **(6 marks)**

**f)** Find the total derivative $\frac{dz}{dt}$ when $z = x^2 + 3xy + 5y^2$ where x = sin t and y = cos t **(4 marks)**

**g)** Find the first five terms of the Taylors series generated by $f(x) = 1/x$ at x = 2 **(4 marks)**

**Question Two**

**a)** Verify that f(x) is a probability density function satisfying $\int_{-\infty}^{\infty} f(x)dx = 1$ if $f(x) = 0.006x(10 - x)$ for $0 \le x \le 10$ and all other values of x **(4 marks)**

**b)** Find the value of $\frac{df}{dt}$ at t = 0 if f(x, y, z) = xy + z and x = cos t, y = sin t and z = t **(5 marks)**

**c)** Given that $f(x) = x + \frac{1}{x}, \ \left[1/2, 2\right]$ satisfy the hypothesis of the mean value theorem in the interval, determine the value of C **(4 marks)**

**d)** Represent $f(x) = \sin x$ as the sum of the Taylor series centred at $\pi/3$ **(7 marks)**

**Question Three**

a) Evaluate $\lim_{x \to \infty} \left\{ \frac{5x^2 + 8x - 3}{3x^2 + 2} \right\}$ **(3 marks)**

b) Find the area enclosed by the line y = x – 1 and the parabola $y^2 = 2x + 6$ **(4 marks)**

c) Test the series below for convergence or divergence $\sum_{k=1}^{\infty} \frac{3^k k^2}{k!}$ **(4 marks)**

d) Find the sum of the geometric series $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + ....$ **(3 marks)**

e) Using Maclaurins series find the first five terms of the expansion $f(x) = e^{3x}$ **(6 marks)**

**Question Four**

a) Define the following terms:
   (i) A bounded sequence **(2 marks)**
   (ii) Increasing sequence **(2 marks)**

b) Sketch the curves with polar equation $r = 2\cos\theta$ and $r = \cos 2\theta$ on the same axes and hence find a Cartesian equation of the curve **(6 marks)**

c) State the indeterminate form arising from:

   (i) $\lim_{x\to\infty} \frac{x}{e^x}$ **(2 marks)**

   (ii) $\lim_{x\to 1} \left\{ \frac{\ln x}{x-1} \right\}$ hence apply L' Hospital's rule to evaluate the given limits **(3 marks)**

d) Sketch and find the volume of the solid obtained by rotating the region bounded by $y = x^3,\ y = 8,\ x = 0$ about the y-axis **(5 marks)**

**Question Five**

a) Evaluate the triple integral:

$$\int_0^1 \int_0^{x^2} \int_{xy}^{x+y} (xyz)\ dz\ dy\ dx$$

**(4 marks)**

b) Find the length of the arc of the curve $x = t^2,\ y = t^3$ that lies between the points (1, 1) and (4, 8) **(5 marks)**

c) Find the rectangular co-ordinates of the given polar points:

   (i) $\left(3, \pi/6\right)$ **(2 marks)**

   (ii) $\left(\sqrt{2}, -\pi/4\right)$ (2 marks)

d) Given $u(x, y) = \ln\left(1 + xy^2\right)$

   show that

$$2\frac{\partial^2 y}{\partial x^2} + y^3 \frac{\partial^2 y}{\partial y \partial x} = 0$$

(5 marks)

e) Find the general formula for the following sequence $2, 4/3, 8/5, 16/7, 32/9$ **(2 marks)**