TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE IN:

BACHELOR OF SCIENCE IN CIVIL ENGINEERING
BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL & AUTOMOTIVE ENGINEERING

SMA 2172: CALCULUS I

END OF SEMESTER EXAMINATION
SERIES: AUGUST 2014
TIME ALLOWED: 2 HOURS

<u>**Instructions to Candidates:**</u>
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Find $(f \circ g)^{-1}(x)$ if $f(x) = 3x - 4$ and $g(x) = 4 - x^2$ **(5 marks)**

b) Given that $y = \dfrac{\sin^2 x}{\ln x}$ find $\dfrac{dy}{dx}$ **(4 marks)**

c) Evaluate the following limits **(3 marks)**

(i) $\displaystyle \lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$ **(3 marks)**

$$\lim_{t \to \infty} \frac{t^2 - 5t - 9}{2t^4 + 3t^3}$$

(ii) **(3 marks)**

$$f(x) = \frac{1}{x-2}$$

d) Find the range and domain of **(3 marks)**

e) Find the derivative of the following function using first principles:

$$f(x) = 2x^2 - 16x + 35$$

(4 marks)

$$V(t) = 2t^2 - 16t + +35$$

f) Suppose that the amount of water in a holding tank at t minutes is given by .
Determine whether:

    (i) The volume of water in the tank is increasing or decreasing at t = 1 minute. **(2 marks)**

    (ii) The volume of water in the tank is changing faster at t = 1 or t = 5 minutes.

(3 marks)

$$\frac{4x^{10} - 2x^4 + 15x^2}{x^3} \, dx$$

g) Evaluate **93 marks)**

Question Two

$$x^2 + xy + y^2 = 7$$

a) Find the equation of tangent to the curve at the point (1, 2) **(4 marks)**

$$x = 80t \qquad y = 64t - 16t^2 \qquad \qquad \frac{dy}{dx} = 0$$

b) Given that and . Find the value of t for which **(4 marks)**

c) Differentiate with respect to x:

$$y = \ln\left(\frac{x+1}{\sqrt{x-2}}\right)$$

    (i) **(4 marks)**

$$y = x^{\sqrt{x}}$$

    (ii) **(4 marks)**

$$3\sqrt{27.05}$$

d) Use differentials to approximate **(4 marks)**

Question Three

a) **(i)** State the THREE conditions for a function to be continuous at a point x = a. **(3 marks)**

$$f(x) = \frac{|x|}{x}$$

    (ii) Show that is not continuous at x = 0 **(4 marks)**

$$f(x) = 6x^5 + 33x^4 - 3x^3 - 100$$

b) Determine all the critical points for the function **(5 marks)**

c) A water tank has the shape of an inverted line with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2m^3$/min, find the rate at which the level is rising when the water is 3m deep. **(5 marks)**

$$y = \sqrt{1 - x^2}$$

d) Differentiate with respect to x **(3 marks)**

Question Four

a) Find $\dfrac{dy}{dx}$ if:

   (i) $y = \dfrac{u-1}{u+1}$ and $u = x^2$ **(4 marks)**

   (ii) $y = 6\dfrac{x^2 - 1}{x^4 + 5x + 1}$ **(3 marks)**

   (iii) $y = (x^3 - 2x + 3)(4x^2 + 1)$ **(3 marks)**

b) Find $(f \circ g \circ h)(x)$ given that $f(x) = \sqrt{x-1}$ $g(x) = x^2 + 2$ and $h(x) = x + 3$. Hence find the range of $f \circ g \circ h(x)$ **(5 marks)**

c) Find the area enclosed by the curve $y = 3x^2 + 2$ the x-axis and the line at x = 3 and x = 5. **(5 marks)**

Question Five

a) A cylindrical can is to be made to hold 1l of oil. Find the dimensions that will minimize the cost of the metal to manufacture. **(6 marks)**

b) Show that $\dfrac{d}{dx}(a^x) = a^x \ln a$ **(4 marks)**

c) Find $\dfrac{dy}{dx}$ from first principles if $y = \dfrac{dx}{\sqrt{x}}$ from first principles if $y = \sqrt{x}$ **(6 marks)**

d) Define the following terms:
   (i) Function **(2 marks)**
   (ii) Monotonic function **(2 marks)**