TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING
BACHELOR OF SCIENCE IN CIVIL ENGINEERING

SMA 2471: NUMERICAL ANALYSIS

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Define an interpolating polynomial **(1 mark)**

b) Find a polynomial of degree three or less to approximate $f(x) = \sin x$ near $x_o = 0$ and use it to approximate sin (0.1) correct to 4dp **(6 marks)**

c) Find the root of $f(x) = \cos x - xe^x$ using the Newton-Raphson's iterative method if $x_o = 1$ correct to 3 dp up to the third step **(6 marks)**

d) Use the trapezoidal rule with 4 intervals to evaluate $\int_1^3 \frac{2}{\sqrt{x}}\,dx$ correct to3 dp **(4 marks)**

e) Using the central difference, obtain a numerical approximation for the second derivative of:

$\log_{10} x$

at x = 5 given h = 0.125 **(4 marks)**

f) Find the unique quadratic polynomial of degree two or les such that f(0) = 1, f(1) = 3 and f(3) = 55 using the Lagrange interpolation **(5 mars)**

g) Determine the value of y when x = 0.1 using Euler's modified method given that y(0) = 1 if $\frac{dy}{dx} = y + x^2$

and h = 0.05 **(4 marks)**

Question Two

$f(x) = \sin x$

a) Determine the step size h to be used in tabulation of in the interval (1, 3) so that a linear interpolation is correct to 4dp **(7 marks)**

b) Determine the volume of a solid generated by revolution where radius r(x), is the perpendicular distance from the x-axis is if:

x	0	1	2	3	4	5	6
r(x)	6.2	5.8	4.0	4.6	5.0	7.6	8.2

Using Simpson's rule with n = 3 and h = 1 **(6 marks)**

$\int_0^{\pi/2} \sin x dx$

c) Evaluate using the trapezoidal rule with n = 10 **(7 marks)**

Question Three

a) Use Simpson's rule to evaluate an approximate value of:

$\int_2^4 \sqrt{1+x}\ dx$

with a step size h = 0.5 correct to 4 significant figures **(4 marks)**

b) Approximate y(0.6) using the Adams bash forth predictor-corrector method with h = 0.1 for the equation $\frac{dy}{dx} = -2xy$ if: **(6 marks)**

x	0.0	0.1	0.2	0.3	0.4
y	1.00	0.9900	0.6908	0.9139	0.8522

c) Evaluate $\int_0^1 \frac{dx}{1+x}$ correct to 4 significant figures using the Gauss Legendre 3 point formula for the interval (-1, 1) **(5 marks)**

d) By Taylor's series of y(x) find y(0.1) correct to 4 decimal places if y(x) satisfies $y' = x - y^2$ and y(0) = 1 **(5 marks)**

Question Four

a) Use a finite difference table to detect the error in the given data hence correct the value:

x	5	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0

f(x)	125.000	132.651	140.608	148.877	157.446	166.375	175.616	185.193	195.112	205.379	216.006

(4 marks)

b) Find an approximate value for sin 0.15 by Lagrange linear interpolation given $\sin 0.1 = 0.09983$ and $\sin 0.2 = 0.19867$

(4 marks)

$$\frac{dy}{dx} = y - \frac{2x}{y}$$

c) Use Taylor's series method to solve and find y(1) given that y(0) = 1

(7 marks)

d) By Newton-Raphson method, find the positive root to the equation $2x^2 + 7x - 6 = 0$ correct to 3 significant figures given $x_0 = 1$ **(5 marks)**

Question Five

$$\frac{dy}{dx} = \frac{t - y}{2}$$

a) Use Euler's method to solve if y(0) = 1 and h = 1 to find y(2) **(4 marks)**

$$\frac{dy}{dx} = y - x$$

b) Apply the 2nd order Runge-Kutta method to find y(0.2) if where h = 0.1 correct to 4 decimal places given y(0) = 2 **(5 marks)**

$$\int_0^{\pi/3} \sqrt{1 - \frac{1}{3}\sin^2\theta}\ d\theta$$

c) Use Simpson's rule to evaluate using 6 intervals **(6 marks)**

d) Given:

x	30°	60°	90°
Cos x°	0.866	0.500	0.000

Find cos 50° using the quadratic Newton forward difference interpolating polynomial

(5 marks)