TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN CIVIL, ELECTRICAL AND MECHANICAL ENGINEERING

(BSCE/BEME/BSEE)

SMA 2271/SMA 2278: ORDINARY DIFFERENTIAL EQUATIONS

END OF SEMESTER EXAMINATION

SERIES: APRIL 2014

TIME ALLOWED: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Explain what is meant by the phrase "complete solution" of a differential equation. **(1 mark)**

$$\frac{dy}{dx} = \frac{y^2 - 1}{x}$$

b) Solve the differential equation **(4 marks)**

c) Solve the linear fractional equation: **(5 marks)**

$$\frac{dy}{dx} = \frac{x+y-3}{x-y-1}$$

(5 marks)

d) Using Laplace transform solve $\frac{dx}{dt} + 2x = 4e^{3t}$ at t = 0 if x = 1 **(5 marks)**

e) Solve using the method of exact differential equation $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

(5 marks)

f) Using the D-operator method find the complete solution if $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$

g) Identify all regular singular points of $(x^3 - 3x^2 + 2x)\frac{d^2y}{dx^2} + (x-2)x\frac{dy}{dx} + 4x^2y = 0$

(5 marks)

Question Two

a) (i) Solve the equation $y'' + a^2y = 0$ where a is a constent by letting $y = e^{mx}$ **(4 marks)**

(ii) Show that the equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ has two linearly independent solution of the form e^{ax}

(3 marks)

b) Solve the Bernoulli's equation of the form $\frac{dy}{dx} - \frac{1}{2}\left(1 + \frac{1}{x}\right)y = \frac{3}{x}y^3$ **(6 marks)**

c) Use Laplace transform to solve the IVP

$$x'' - 3x' + 2x = 2e^{3t}$$
$$x_0 = x(0) = 5$$
$$x_1 = \frac{d(x0)}{dt} = 7$$

(7 marks)

Question Three

a) Solve the equation $(D^2 + D - 2)y = 2x - 40\cos 2x$ by the method of undetermined coefficient.

(10 marks)

b) Find the singular points of the differential equation and determine whether they are regular or ordinary points:

$$x^2(1-x)y'' + (1-x)y' + y = 0$$

(5 marks)

c) Solve the initial value problem:

$$(x^2+9)\frac{dy}{dx}+xy=0;\ \ y(0)=1$$

(5 marks)

Question Four

a) State THREE reasons why the differential equation below is non-linear. **(3 marks)**

$$x\frac{d^2y}{dx^2}+\left(x\frac{dy}{dx}-y\right)^2-3y^2=0$$

b) (i) Show that $ydx-xdy=0$ is not exact. **(2 marks)**

(ii) Show that $\frac{1}{x^2}$ is an integrating factor for the equation in b (i) above **(3 marks)**

(iii) Solve the equation $ydx-xdy=0$ using the integrating factor method. **(4 marks)**

c) (i) Show that $\frac{xy}{x^2+y^2}$ is a homogeneous function in x and y **(2 marks)**

(ii) Using the substitution y = vx transform the equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ into an equation containing v and x only **(3 marks)**

(iii) Hence solve the resulting equation using the method of separation of variables in (ii) above. **(3 marks)**

Question Five

a) Solve $yy''=(y)^2$ by reducing the order, by substitution $y'=p$ and $y''=p\frac{dp}{dy}$ **(4 marks)**

b) Given the differential equation:

$$(x^2-1)\ \frac{d^2y}{dx^2}+3x\frac{dy}{dx}+xy=0$$

(i) Normalize the equation **(1 marks)**

(ii) Show that x = 0 is an ordinary point for the equation **(2 marks)**

c) An electric circuit has a constant electromotive force E= 40v, a resistor of 10 Ω and an inductance 0.2 Henry, with initial current 0 at t = 0 and basic differential equation is:

$$L\frac{di}{dt}+Ri=E$$

Determine the steady current after a long time.

d) Use the inverse D operator method to solve $(D^2 - 5D + 6)\,y = e^x \cos 2x$ **(8 marks)**