TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING/BUILDING & CIVIL ENGINEERING/CIVIL ENGINEERING
(BSME 13/BSCE 13/BSCE 13)

SMA 2177: APPLIED GEOMETRY

END OF SEMESTER EXAMINATION
SERIES: AUGUST 2013
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) The point $P\left(\frac{\sqrt{3}}{2}, y\right)$ is on the unit circle, centre the origin and in the fourth quadrant. Find its y-coordinate. **(2 marks)**

b) The commander of a coastal base A, situated 6.3km north of a small base B sights a ship at C which is 7.3km from A and 060° from B. Find the distance of the ship from B using the cosine rule. **(5 marks)**

c) Prove the identities **(2 marks)**

$$\frac{\sec t - 1}{t \sec t} = \frac{1 - \cos t}{t}$$

d) Find the equation of the line perpendicular to the line $3x = 2y + 6$ and passing through the mid-point of the x and y intercepts. Express your answer in the form $ax + by + c = 0$ where a, b, c are integers. **(6 marks)**

e) Given that $\sin \alpha = 2/3$ and $\sin \beta = 1/3$ and that α and β are between 0 and $\pi/2$, evaluate $\sin(\alpha + \beta)$ leaving your answer in simplified surd form.

f) Sketch the curves defined by $r = \cos\theta + 1$ and $r = \cos\theta - 1$ using the same initial ray. What would you conclude about the two equations? **(6 marks)**

g) Find the following for the graph:

$$y = 3\sin\left(2x + \frac{\pi}{2}\right) + 1$$

(i) Amplitude
(ii) Period
(iii) Phase shift **(4 marks)**

SECTION B (Answer any TWO questions from this section)

Question Two

a) Given the hyperbola $9x^2 - 16y^2 = 144$ find:
(i) The vertices, the foci and the asymptotes.
(ii) Graph the hyperbola **(10 marks)**

b) Describe and sketch the curve which is the locus of the point p(x, y) such that the ratio of its distance from the point (3, 0) to its distance from the line $x = \frac{25}{3}$ is equal to $3/5$ **(6 marks)**

c) For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $\overline{SP} + \overline{S'P} = 2a$ where S and S' are the foci. **(4 marks)**

Question Three

a) Show that the distance of a point P(x, y) from the line $ax + by + c = 0$ is given by $d = \dfrac{|ax_1 + by_1 + c_1|}{\sqrt{a^2 + b^2}}$ **(10 marks)**

b) Find the equation of the circle inscribed in the triangle whose sides are the lines $4x + 3y = 24,\ 3x - 4y = 18,\ 4x - 3y + 32 = 0$ **(10 marks)**

Question Four

a) Find the coordinates of the points where the line $4x - 5y + 69 = 0$ cuts the curve given parametrically by $(at2,\ 2at)$ **(5 marks)**

b) Given the parabola $y^2 - 2y - 8x - 31 = 0$ find the vertex, the focus and the directrix. **(6 marks)**

c) If $\sin(x + \alpha) = \cos(x - \beta)$, find $\tan x$ in terms of α *and* β **(3 marks)**

d) Show that the tangent to the circle $x^2 + y^2 = 13$ at the point $(-2,3)$ touches the circle $x^2 + y^2 - 10x + 2y - 26 = 0$ **(6 marks)**

Question Five

a) The normal at a point $P(4\cos\theta,\ 3\sin\theta)$ on an ellipse, centre (0, 0) meets the x and y axes at A and B. Show that the locus of M, the mid-point of AB is an ellipse with the same eccentricity as given ellipse. Sketch the two ellipses on the same set of axes. **(12 marks)**

b) Show that if $t = \tan \frac{1}{2}\theta$, then $\sin\theta = \dfrac{2t}{1+t^2}$ **(4 marks)**

c) Find the slope of the line bisecting the angle from L_1 with slope 7 L_2 with slope 1 **(4 marks)**