TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

DIPLOMA IN MECHANICAL ENGINEERING
(PLANT, AUTOMOTIVE & PRODUCTION OPTIONS)

AMA 2302: ENGINEERING MATHEMATICS VI

END OF SEMESTER EXAMINATION
SERIES: APRIL 2013
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination

- *Answer Booklet*
- *Mathematical Table*
- *Scientific Calculator*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**

Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) (i) Show that $\underset{\sim}{A} \bullet \underset{\sim}{B}$ is a scalar quantity…. **(3 marks)**

$$\underset{\sim}{r_1} = 2\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k} \qquad \underset{\sim}{r_2} = 6\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$$

(ii) Find the scalar product of $\underset{\sim}{r_1}$ and $\underset{\sim}{r_2}$ **(2 marks)**

(iii) Hence determine the angle between $\underset{\sim}{r_1}$ and $\underset{\sim}{r_2}$ **(3 marks)**

b) Evaluate the following integrals:

(i) $\int_{-3}^{3}\int_{0}^{1}\int_{1}^{2}(x+y+z)dx\ dy\ dz$ **(5 marks)**

(ii) $\int_{0}^{\pi}\int_{0}^{a\sin\theta} r\ dr\ d\theta$ **(3 marks)**

c) Find the area bounded by the parabola $y = x^2$ and the line $y - 2x - 3 = 0$ Hence sketch the area graph. **(4 marks)**

d) Five balls are drawn from a bag containing six red and four black balls. Determine the probability that:

(i) 3 are red and 2 are black **(2 marks)**

(ii) All five are red balls **(2 marks)**

e) The probability that a person suffers bad reaction from an injection of a given serum is $\frac{1}{1000}$. Use Poisson distribution to determine the probability that out of 2000:

(i) Exactly three suffer bad reaction **(3 marks)**

(ii) More than 2 people suffers bad reaction **(3 marks)**

SECTION B (Answer any TWO questions from this section)

Question Two

$$\underset{\sim}{x} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k},\ \underset{\sim}{y} = a\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k} \qquad \underset{\sim}{z} = \underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$$

a) (i) Three coplanar vectors are: and
Determine the value of a **(3 marks)**

$$\underset{\sim}{A} = 2\underset{\sim}{i} - 5\underset{\sim}{j} + 7\underset{\sim}{k},\ \underset{\sim}{B} = 3\underset{\sim}{i} + 8\underset{\sim}{j} - \underset{\sim}{k}$$

(ii) Given the vectors

Determine:

(I) $\underset{\sim}{A} \bullet \underset{\sim}{B}$ **(2 marks)**

(II) $\underset{\sim}{A} \times \underset{\sim}{B}$ **(2 marks)**

(III) The angle between $\underset{\sim}{A}$ and $\underset{\sim}{B}$ **(2 marks)**

b) Given that $\underset{\sim}{F} = x^2 yz\,\underset{\sim}{i} + xyz\,\underset{\sim}{j} + y^2 z\,\underset{\sim}{k}$ and $\phi = xyz - 2y^2 z + x^2 z^2$

$div\,curl F = curl\,grad\,\phi = 0$

(i) Show that **(7 marks)**

(ii) Determine $div\,grad\,\phi$ at (1,1,1) **(4 marks)**

Question Three

a) Evaluate the following integrals:

(i) $\int_0^1 dx \int_0^x dy \int_0^y dz$ **(4 marks)**

(ii) $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy\ dx}{1+x^2+y^2}$ **(4 marks)**

(iii) $\iiint_s xyz\ dx\ dy\ dz$ where $s(x,y,z) = x^2 + y^2 + z^2 = 1$, $x \geq 0,\ y \geq 0,\ z \geq 0$ **(7 marks)**

b) Determine the volume of a solid bounded by the following surfaces using triple integral:

$Z = 0,\ x^2 + y^2 = 1,\ x + y + z = 3$

(7 marks)

Question Four

$A = x^3 y\,\underset{\sim}{i} + (x+z)y\,\underset{\sim}{j} + x^2 z^2\,\underset{\sim}{k}$ and $\phi = 2x^2 y + xyz - 4y^2 z^2 - 5$

a) If

Determine at (1, 1, 3)

(i) Div A **(3 marks)**

(ii) Grad ϕ **(3 marks)**

(iii) Curl A **(3 marks)**

b) Given two vectors $A = 3\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k},\ B = \underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ show that $A \times B$ is perpendicular to the vector. $C = 9\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$

(3 marks)

c) (i) If $f(x, y, z) = xyz - 2y^2z + x^2z^2$ determine $divgradf(x, y, z)$ at a point $(2,4,1)$ **(3 marks)**

(ii) Determine a unit normal to the surface $\phi = 4xz^3 - 3x^2y^2z$ at a point (2, -1,2)

(4 marks)

Question Five

a) The weekly wages of 1000 workmen are normally distributed around a mean of ksh 70 with standard deviation of ksh 5. Estimate the number of workers whose wages will be:

(i) Between ksh 69 and ksh 72 **(4 marks)**

(ii) Less than ksh 69 **(4 marks)**

(iii) More than ksh 72 **(2 marks)**

b) (i) If r has a binomial distribution $P(r) = n_{cr} q^{n-r} pr$ show that the mean u is given by np

(7 marks)

(ii) Fit Poisson distribution to the following data:

x	0	1	2	3	4
f	192	100	24	3	1

(3 marks)