TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF TECHNOLOGY IN RENEWABLE ENERGY & ENVIRONMENTAL PHYSIC

BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS

APS 4106: WAVES & VIBRATION

END OF SEMESTER EXAMINATION

SERIES: APRIL 2014

TIME ALLOWED: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **FOUR** printed pages

Question One (Compulsory)

$$\frac{d^2y}{dx^2} = -k^2y$$

a) A simple harmonic motion can be described by a differential equation of the form where all the terms have their usual verify that the equation has solution of the form $y = A\cos(kx) + B\sin(kx)$

(3 marks)

b) A mass at the end of a spring oscillates with an amplitude of 5cm at a frequency of 1Hz (cycles per second). At t = 0 the mass is at its equilibrium position (x = 0)

(i) Find the possible equation describing the position of the mass as a function of time in the form $x = A\cos(wt + \alpha)$

What are the numerical values of A, w and α ? **(6 marks)**

(ii) What are the values of x, $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$ at $t = 8/3$ sec ? **(3 marks)**

c) An object of mass 0.2kg is suspended from a spring whose spring constant is 80N/m. The body is subject to a resistive force given by –bv where v is the velocity (m/sec) and b = $4Nm^{-1}$ sec:

(i) Set up the differential equation of motion for free oscillations of the system and find the period of such oscillation **(4 marks)**

(ii) The object is subjected to a sinusoidal force given by $F(t) = Fo\sin wt$, where F_o = 2N and w 30 sec^{-1}. In the steady state, what is the amplitude of the forced oscillation? **(2 marks)**

(iii) Instead of a hiring force (in part (ii)), we now oscillate the end of the spring at the top end vertically with a harmonic displacement $X = X_o \sin(wt)$. Set up the differential equation of motion for this driven oscillator. **(4 marks)**

d) A generator of EMF $V(t) = V_o \cos wt$ is connected in series with resistance R_1 an inductance L and a capacitance C.
A coin is tossed 3 times. Let X be the random variable denoting the number of heads observed.

(i) Write down the differential equation for the current I in the circuit and for the charge q1 on the capacitor **(4 marks)**

(ii) Solve for q (w, t) **(2 marks)**

(iii) Solve for I (w; t) **(2 marks)**

Question Two

The figure below shows a pulse on a string of length 100m with fixed ends. The pulse is traveling to the right without any change of shape at a speed of 40m/sec

D.M

a) Make a clear sketch showing how the transverse velocity of the string varies with distance along the string at the instant when the pulse is in the position shown. **(6 marks)**

b) What is the maximum transverse velocity of the string (approximately) **(6 marks)**

c) If the total mass of the string is 2kg, what is the tension T in it? **(3 marks)**

d) Write down an equation for y(x, t) that numerically describes sinusoidal waves of wavelength 5m and amplitude 0.2m travelling in the negative x-direction on a very long string made of the same material and under the same tension as above. **(5 marks)**

Question Three

a) Two vibrations along the same line are described by the equations:

$$x_1 = a \sin w_1 t$$

$$x_2 = a \sin w_2 t$$

(i) Find the beat period of the disturbances. **(6 marks)**
(ii) Draw a careful sketch of the resultant disturbances **(4 marks)**

b) Two vibrations at right angles to one another are described by the equations:

$$x = a_1 \sin(wt + \phi)_1$$

$$y = a_2 \sin(wt + \phi_2)$$

$$\phi_2 - \phi_1 = \pi/2$$

Show that if $a_1 = a_2$, and then the projection of the particle will be a circle whose radius is a_1. **(10 marks)**

Question Four

$$X = A\sin(w + +\phi)$$

a) Consider a particle attached to a spring executing a motion with A = 0.32m. At t = 0, it is at x = -0.07m and a velocity -2m/s. The total energy is 5.6J. Find:

ϕ

(i) **(3 marks)**
(ii) The frequency + **(3 marks)**
(iii) The spring constant K **(3 marks)**
(iv) The mass of the particle **(3 marks)**

b) The displacement from equilibrium, S(t) of the pen of a chart recorder can be modeled as a damped harmonic oscillator satisfying the homogenous differential equation:

$$\ddot{S}(t) + y\dot{s}(t) + w_0^2 s(t) = 0$$

(i) Find the time evolution of the displacement if the pen is critically damped and subject to the initial condition s (t = 0) and s (t = 0) = Vo **(4 marks)**
(ii) Show the plot of the critically damped system **(4 marks)**

Question Five

a) Construct the Lizzojons figures for the following motions.

(i) $x = \cos 2w+, \quad y = \cos w+$ **(2 marks)**

(ii) $x = \cos\ 3w+, \quad y = \cos\left(wt - \pi/4 \right)$ **(4 marks)**

b) A transverse travelling wave on a cord is represented by D = 0.485m (5.6x + 84t) where D and x are metres and t in seconds. For this wave, determine;

(i) The wavelength **(2 marks)**

(ii) Frequency **(2 marks)**

(iii) Velocity (magnitude and direction) **(2 marks)**

(iv) Amplitude **(2 marks)**

c) A 440-Hz longitudinal wave in air has a speed of 345m/s

(i) What is the wavelength **(1 mark)**

(ii) How much time is required for the phase to change by 90^o at a given point in space? **(3 marks)**

(iii) At a particular instant, what is the phase difference (in degrees) between two points 4.4.cm apart? **(2 marks)**