TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN MATHEMATICS & COMPTUER SCIENCE

AMA 4323: ORDINARY DIFFERENTIAL EQUATIONS II

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **TWO** printed pages

Question One (Compulsory)

a) (i) State the existence and uniqueness theorem for an nth order linear differential equation **(3 marks)**

(ii) Prove that the equation:

$$2\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} + 3x^2\frac{dy}{dx} - 5y = \sin x$$

$$y(4) = 3 \quad y^{11}(4) = -7/2$$

(3 marks)

has a unique solution

b) (i) Prove that $y = e^{2x}$ is a solution of $(2x+1)\frac{d^2y}{dx^2} - 4(x+1)\frac{dy}{dx} + 4y = 0$ **(2 marks)**

(ii) Find a linearly independent solution of the above equation by reducing the order **(5 marks)**

(iii) Hence write the general solution of the equation **(1 mark)**

$$yy''=(y')^2$$

c) Solve the non-linear equation (4marks)

d) Use the Rodriguez formula for Legendre to find the polynomial for $P_1(x)$ and $P_2(x)$ **(5 marks)**

e) (i) Verify that the equation $yz\ dx - z^2 dy + xydz = 0$ is exact **(2 marks)**

(ii) Hence find the solution of the equation in (i) above **(5 marks)**

Question Two

a) Locate and classify the singular points of the equation:

$$x^4 - 2x^3 + x^2 \frac{d^2 y}{dx^2} + 2(x-1)\frac{dy}{dx} + x^2 y = 0$$

(7 marks)

b) Find the power series of $1 - x^2 y'' - 2xy' + 2y = 0$ about x = 0 **(13 marks)**

Question Three

a) (i) Verify the condition of integrability of the equation:

$$(z+z^3)\cos dx - (z+z^3)dy(1+z^2)(y-\sin x)dz = 0$$

(3 marks)

(ii) Hence solve the above equation **(5 marks)**

b) Solve the following Bessel's equation up to the x^4 term

$$x^2 y'' + xy' + (x^2 - p^2 y) = 0$$

(12 marks)

Question Four

a) Solve the following equation by transforming to normal form:

$$y'' + \left(2 + \frac{4}{3}^{x}\right)y' + \frac{1}{9}(24 + 12x + 4x^2)y = 0$$

(5 marks)

b) Find the general solution to $2t^2 - y^{11} t y^1 - 3y = 0$ given that $y_1(t) = t_{-1}$ is a solution by the method for reducing the order **(5 marks)**

c) Find the power series of the following Legendre's differential equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0$

(10 marks)

Question Five

a) Solve $(3x+2)^2 y''+3(3x+2)y'-36y = 3x^2+4x+1$ **(10 marks)**

b) The differential equation of a shaft which whirling with the line bearings horizontal is given by:

$$EI\frac{d^4y}{dx^4}-\frac{Ww^2y}{g}=W$$

where W is the weight of the shaft and w is the whirling speed. Taking the length of the shaft as 2L with the origin at it's centre and short bearings at both ends:

$$x=\pm 1, \quad y=\frac{d^2y}{dx^2}=0$$

(i.e for)

$$y = \frac{g}{2u^2}\left[\frac{\cos mx}{\cos mL}+\frac{\cosh mx}{\cosh mL}-2\right] \qquad M^4=\frac{Wm^2}{gEI}$$

Show that where and maximum deflection is $\frac{g}{2w^2}[\sec mL+\sec hmL-2]$

(10 marks)