TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN STATISTICS & COMPUTER SCIENCE
BACHELOR OF MATHEMATICS & COMPUTER SCIENCE
(BMCS/BSSC)

AMA 4217: LINEAR ALGEBRA I

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Consider point P(3, K, -2) and Q(5, 3, 4) in $\Re^3$. Find K so that P Q is orthogonal to the vector U=(4, -3, 2) **(4 marks)**

b) Each of the following equations determines a plane in $\Re^3$. Do the two planes intersect? If so describe their intersection:

$$x_1 + 4x_2 - 5x_3 = 0$$
$$2x_1 - x_2 + 8x_3 = 9$$

(4 marks)

c) Is the set {(1, 0, -1), (0, 1, -1), (-1, 1, 0)} a spanning set for $\Re^3$? Justify your answer **(4 marks)**

d) Reduce the following matrix to reduced echelon form:

$$\begin{pmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{pmatrix}$$

(4 marks)

e) Find the values of λ so that $\vec{a} = 3i - 2\lambda j + 2k$ and $\vec{b} = 2\lambda i + \lambda j + 4k$ are perpendicular **(3 marks)**

f) A subset U of $\Re^4$ is spanned by the set comprising of the vectors:
(1, 2, 0, 4), (2, 1, -1, 3), (0, 3, 1, 5), (2, 4, 0, 8)
(i) Find a basis for U **(3marks)**
(ii) Extend the vectors in (i) above to a basis for $\Re^4$ **(2 marks)**

g) Let $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ be the matrix map A: $\Re^2 \to \Re^2$ give a geometric description of the transformation **(3 marks)**

h) Show that the subset $U = \{(x, y, z) \in \Re^3 | 2x + 3y - z = 0\}$ is a subspace of $\Re^3$. What does it represent geometrically. **(3 marks)**

Question Two

a) Let $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$. Show that T is a one to one linear transformation **(4 marks)**

b) Let U = (1, -3, 2) and V = 2, -1, 1) be vectors in $\Re^3$

(i) Write W = (1, 7, 4) as a linear combination of U and V **(4 marks)**

(ii) Extend vectors (U,V) to form a basis for $\Re^3$ **(4 marks)**

c) Let $B = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$
(i) Define the rank of a matrix **(2 marks)**
(ii) Find the rank of matrix B **(3 marks)**

d) Let V be the vector space of 2 x 2 matrices over $\Re$. Determine whether the matrices A, B $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ are linearly independent **(3 marks)**

Question Three

Set $A = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix}, U = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, b = \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, C = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ and define the transformation $T : \Re^2 \to \Re^3$ by $T(x) = Ax$ so that

$$T(x) = Ax = \begin{pmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 & -3x_2 \\ 3x_1 & +5x_2 \\ -x & +7x_2 \end{pmatrix}$$

a) Find T(U) the image of U under the transformation T **(3 marks)**

b) Find x in $\Re^2$ whose image under T is b **(6 marks)**

c) Is there more than one x whose image under T is b? **(2 marks)**

d) Determine if C is in the range of the transformation T **(5 marks)**

Question Four

a) Given that V_1 and V_2 are in the vector space V and let $H = span\{v_1, v_2$ } Show that H is a subspace of V. **(6 marks)**

b) Find a unit vector perpendicular to $\vec{a} = (4,-3,1)$ and $\vec{b} = (2,3-1)$ **(5 marks)**

c) Determine the dimension of the subspace H of $\Re^3$ spanned by the vectors V_1, V_2, and V_3

$$V_1 = \begin{pmatrix} 2 \\ -8 \\ 6 \end{pmatrix} \quad V_2 = \begin{pmatrix} 3 \\ -7 \\ -1 \end{pmatrix} \quad V_3 = \begin{pmatrix} -1 \\ 6 \\ -7 \end{pmatrix}$$

(4 marks)

d) Show that w is not a subspace of V where w consists of all matrices A for which $A^2 = A$

(5 marks)

Question Five

a) Given that $V_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, V_2 = \begin{pmatrix} 5 \\ -4 \\ 7 \end{pmatrix}, V_3 = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 3 \\ n \end{pmatrix}$ for what values of n will y be in the span $\{v_1, v_2, v_3\}$ **(5 marks)**

b) Find the angle between the vectors:

$$\vec{0A} = 4i - 5j + 2k$$

$$\vec{0B} = -i + 2j + 3k$$

To the nearest degree **(5 marks)**

c) Is the subset P $\{a_0 + a_1 x + a_2 x^2 | a_0 + 2a_1 + a_2 = 1\}$ a subspace of polynomials **(5 marks)**

d) Given that U x V = i + 3j Find $U \times (2U + V) + 2V \times (3V - U)$ **(5 marks)**