TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN CIVIL ENGINEERING

SMA 2273: APPLIED MATHEMATICS

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Define the following terms:
- **(i)** Variable speed
- **(ii)** Velocity
- **(iii)** Work
- **(iv)** Power **(4 marks)**

b) Find the tangential acceleration and the normal acceleration of a particle which moves on the ellipse $r = a\cos wt\,\underset{\sim}{i} + b\sin wt\,\underset{\sim}{j}$

(3 marks)

c) A particle of mass 5 units moves along a space curve given by:

$\underset{\sim}{r} = \left(3t^2 + t\right)\underset{\sim}{i} + \left(3t + 2\right)\underset{\sim}{j} + \left(2t^4 - 4t^3\right)\underset{\sim}{k}$

find its:

(i) Velocity **(2 marks)**

(ii) Acceleration **(2 marks)**

(iii) Momentum **(2 marks)**

(iv) Force at t = 1 **(2 marks)**

d) A particle is projected from the ground with a speed of 30m/s at an angle of 30° to the ground. Calculate: (use g = 9.81m/s^2)

(i) Time of flight

(ii) The horizontal range **(4 marks)**

e) A force of magnitude 80N acts along a positive x axis. Another force of magnitude 50N is inclined at an angle 120o to the horizontal surface. Find the resultant force and its direction from 80N force **(4 marks)**

f) Consider the horizontal motion at any time t to be $S_x = U_x t$ and $x = ut\cos x$ show that range is given by:

$$x = \frac{u^2 \sin 2\alpha}{g}$$

(3 marks)

g) A streamer takes time t, to travel distance L up a river, and time t_2 to return. Show that the speed of the streamer with respect to the stream is

$$\frac{L\left(t_1 + t_2\right)}{2t_1 t_2}$$

(4 marks)

Question Two

a) A gun whose mass is 0.8kg fires a bullet whose mass is 0.016kg with a velocity of 700m/s. Compute the velocity of the gun recoil **(3 marks)**

b) A block of mass 5kg is initially at rest on a smooth horizontal plane. A horizontal force of 20N is applied to it for 10 seconds. Find the speed of the block after this time **(3 marks)**

c) Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along a curve $x = t^2 + 1 \quad y = 2t^2 \quad z = t^3$ and from t = 1 second to t = 2 seconds **(4 marks)**

d) A particle of mass 2 units moves a long a space curve defined by $\vec{V} = \left(4t^2 - t^3\right)\hat{i} - 5tj + \left(t^4 - 2\right)\hat{k}$. Find:

(i) The momentum

(ii) The force acting on the particle at time t = 1 second **(3 marks)**

e) (i) Differentiate between momentum of a body and impulse **(2 marks)**

(ii) Find the impulse, induced by a force given by $\overline{F} = \sin t\,\hat{i} + \cos t\,\hat{j}$ over a duration $0 \le t \le \pi/2$ **(3 marks)**

Question Three

$$F = 4\left(1 - e^{-t/6}\right)$$

a) A particle of mass 8kg is acted upon by a force . If the body is initially at rest. Find the velocity of the particle after 3 second **(3 marks)**

b) A particle of unit mass moves from rest along a straight line under a force (2 – 0.1V)N where V is the velocity in m/s. Find the displacement when V is 10m/s **(3 marks)**

c) A particle is projected from a point O on a horizontal plane with speed 40m/s at an angle θ to the horizontal where $\tan\theta = 4/3$. Find:

(i) Time taken for P to return to the horizontal plane **(2 marks)**

(ii) The maximum height attained and horizontal range **(2 marks)**

(iii) The speed of P after 2.2 seconds **(3 marks)**

d) Show that the acceleration of a particle which travels along a curve with velocity $\vec{V}$ is given by $\vec{a} = \frac{dv}{dt}\overline{T} + \frac{V^2}{R}\overline{N}$ where $\overline{T}$ is a unit tangent vector to the space curve, $\vec{N}$ is the unit principal normal and R is the radius of curvature **(4 marks)**

e) A force of magnitude 4N and 3N acts along sides AB and AD of a square ABCD respectively with sides 2m. Find the perpendicular distance of the line of action of their resultant force R **(3 marks)**

Question Four

a) State the first and the second Newton's Laws of Motion **(4 marks)**

b) A body of mass M falls from rest through a height h and is brought to rest by penetrating through a depth a depth d into some soil. Show that the average resistance of the soil is $mg\left(1 + \frac{h}{d}\right)$ **(6 marks)**

c) Find total work done in moving a particle in a force field given by $\underset{\sim}{F} = 3xy\underset{\sim}{i} - 5z\,\underset{\sim}{j} + 10x\,\underset{\sim}{k}$ along the curve $x = t^2 + 1, \quad y = 2t^2 \quad z = t^3$ from t = 1 to t = 2 **(5 marks)**

d) A man can swim directly across a stream of width 5 metres in t seconds when there is no current and $S\sqrt{\frac{1}{t^2} - \frac{1}{T^2}}\, m/s$ time T where there is current show that the velocity of the current is **(5 marks)**

Question Five

a) A particle of mass 3kg rest on the surface of a rough plane which is inclined at 30° to the horizontal plane. It is connected by a light smooth pulley at the top of the plane to a particle of mass 3kg which is hanging freely. If the coefficient of friction between 2kg mass and the plane is 1/3. Find the acceleration of the system when it is released from rest and the tension in the string. Find also the force exerted by the string on the pulley **(7 marks)**

$$\underset{\sim}{F} = yz\,\underset{\sim}{i} + xz\,\underset{\sim}{j} + xy\,\underset{\sim}{k}$$

b) Show that is a conservative force filed and hence find the potential function associated with this field with zero value at the point (1, 1, 1) **(6 marks)**

c) Define the term conservative force **(2 marks)**

$$\underset{\sim}{F} = -\nabla \underset{\sim}{V}$$

d) If where is a single valued function and has continuous partial derivative. Show that the work done in moving a particle from one point $P_1(x_1, y_1, z_1)$ to another point $P_2(x_2, y_2, z_2)$ is independent of the path joining the two point **(5 marks)**