TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE
BACHELOR OF SCIENCE IN STATISTICS & COMPUTER SCIENCE

AMA 4108: DISCRETE MATHEMATICS

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Write a logically equivalent statement to "if you do not attend recitation, then you are not wise"
(2 marks)

b) Show that the following argument form is valid:

$p \vee (q \vee r)$

$\sim r$

$\therefore p \vee q$

(4 marks)

c) Let A and B be any two sets prove that;

$(A \cap B)^{\subset} = A^{\subset} \cup B^{\subset}$

(5 marks)

using elements argument method

$A = \{0,1,2\},\ B = \{a,b\}, C = \{m,n\}$

d) Let **(3 marks)**

Find $C \times (A \times B)$

e) Prove by mathematical induction that for every positive integer, n **(5 marks)**

$1 + 3 + 5 + \ldots + (2n - 1) = n^2$

f) Find the range of $f : \Re \to R$ defined by $f(x) = \dfrac{3x}{x^2 + 1}$ **(3 marks)**

g) Let P(m, n) be "n is greater than or equal to m" where the universe of discourse is the set of non-negative integers. What is the truth value of $\exists n \forall m P(m, n)$ and $\forall m \exists n P m(, n)$ justify your answers **(4 marks)**

h) Determine whether the statement $(\sim p \vee q) \vee (p \wedge \sim q)$ is a tautology, a contingency or absurdity **(2 marks)**

i) Find $"f \circ g"$ and $"g \circ f"$ where $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{1}{x^2}$ **(2 marks)**

Question Two

a) Show that $(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$ **(6 marks)**

b) Prove by contradiction that $6 - 7\sqrt{2}$ is irrational **(4 marks)**

c) State the converse, inverse the contra positive of the proposition. "If Sara plays her guitar, then Jack will sing" **(3 marks)**

d) Use truth table to determine whether the given argument form is valid or invalid **(5 marks)**

$$
\begin{aligned}
&p \wedge q \\
&p \vee q \\
&p \to r \\
&\therefore r
\end{aligned}
$$

e) Define $f : z \to z$ by the rule $f(n) = n^2; \forall n \in z$ if one to one? **(2 marks)**

Question Three

a) Let A and B be the subsets of U with u(U) = 150, n(A) = 80, n(B) = 55 and $nA(\cup B)^{c} = 30$ Find $n(A \cap B)$

(4 marks)

b) Find the power set of the set $A = \{\phi, 1, 2, (1,2)\}$ **(3 marks)**

c) Prove the following using membership table $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **(5 marks)**

d) Let $f : \Re \to \Re$ defined as $f(x) = x^3 + 5$

(i) Determine whether f is one-to-one and onto **(6 marks)**

(ii) Find the formula that defines the inverse function f^{-1} **(2 marks)**

Question Four

a) Prove by mathematical induction $5^n - 2^n$ is divisible by 3 for any $n \geq 1$ **(7 marks)**

b) Show that the functions $f : \Re \to (1, \infty)$ and $f(x) = 10^{2x} + 1$ and $g(x) = \frac{1}{2}\log_{10}(x-1)$ are inverses of each other. **(7 marks)**

c) Let a relation A on the set of real numbers R be defined as follows:

$\forall a, b \in \Re \quad a\mathrm{A}b \Leftrightarrow a < b$

Determine whether A is relative, symmetric or transitive **(3 marks)**

d) Write the converse, inverse and contra positive of the following statements:

$\sim p \to \sim q$

(3 marks)

Question Five

a) Let $f, g : \Re \to \Re$ be defined by $f(x) = 2x - 3$ and $g(x) = \frac{x+1}{5}$

(i) Find f^{-1} and g^{-1} **(2 marks)**

(ii) $(fog)^{-1} = g^{-1} o f^{-1}$ **(5 marks)**

b) Let $A = (1,2,3)$ and $B = \{1,2,3,4\}$ define a binary relation R from A to B as follows $R = \{(a,b) \in A \times B : a < b\}$

(i) Find the ordered pairs in R **(2 marks)**

(ii) Find domain and range of R **(2 marks)**

c) Construct truth table for the following compound statement **(3 marks)**

$p \to q \leftrightarrow p \wedge q$

(3 marks)

d) Prove the following statement by contraposition "For all integers m and n, if m + n is even, the m and n are both even or m and n are both odd" **(3 marks)**

e) Prove that $9n^2 + 3n - 2$ is even **(3 marks)**