# TECHNICAL UNIVERSITY OF MOMBASA

# Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

**DEEE III/DEPE III/ DTIE III/ DICE III/DEAE III**

AMA 2203: ENGINEERING MATHEMATICS III

**END OF SEMESTER EXAMINATION**
**SERIES:** APRIL 2013
**TIME:** 2 HOURS

**Instructions to Candidates:**
You should have the following for this examination
- *Answer Booklet*
- *Non-programmable Scientific Calculator*
- *Mathematical Table*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown
This paper consists of **FIVE** printed pages

**SECTION A (COMPULSORY)**

**Question One**

**a)** (i) A box with sides of length x, y, z mm is expanding along the x and y sides at a rate of 2 and 3mm per second but contracting along the z side at a rate of 4mm per second. Find the rate of change of volume when x = y = 10mm, z = 20mm. **(3 marks)**

**(ii)** If $V = \angle n\left(x^2 + y^2\right)$ prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ **(5 marks)**

**b)** (i) Find all first and second partial derivatives for $z = 3x^2 2xy + 4y^2$ **(3 marks)**

(ii) Given that volume of cylinder is given by $V = \pi\tau^{2h}$ where $r = 5cm,\ h = 10cm.$ Find the appropriate increase in volume when r increase by 0.2cm and h decreases by 0.1cm. **(4 marks)**

**c)** (i) Find the differential $\frac{dy}{dx}$ and its value at $x = 2$ if $y = 3x^4 - 7x^3 + 4x^2 + 3x - 4$. Find the second order $\frac{d^2 y}{dx^2}$ of the same expression of y. **(2 marks)**

**(ii)** Find the general solution of $\tan x \frac{dy}{dx} = y$. Also find the particular solution for which y = 2 at $x = \frac{1}{6}\pi$

**(4 marks)**

**d)** (i) solve $\frac{dT}{d\theta} = \mu T = 0$ given that T = $T_o$ where $\theta = 0$ **(2 marks)**

(ii) A constant e.m.f. E is introduced into an (L, R) circuit. If "i" is the current at the instant t in a given direction caused by E, there is an e.m.f $\angle \frac{di}{dt}$ in the reverse direction due to the inductance L. Obtain the differential equation of the circuit and hence find the current i at time t. **(3 marks)**

**e)** Form the augmented matrix of the following set of equations and solve by elimination:

$$x_1 - 4x_2 - 2x_3 = 21$$
$$2x_1 + x_2 + 2x_3 = 3$$
$$3x_1 + 2x_2 - x_3 = -2$$

**(4 marks)**

**SECTION B (Answer any TWO questions from this section)**

**Question Two**

**a)** (i) If $I = \frac{V}{R}$ and V = 250 volts and R = 50,0hms, find the change in I resulting from an increase of 1 volt in V and an increase of 0.5,0hm in R. **(2 marks)**

(ii) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $z = (3x + 2y)(4x - 5y)$

**(2 marks)**

**b)** (i) Find all first and second partial derivatives for $z = \frac{x + y}{x - y}$ **(4 marks)**

(ii) If $z = \angle n\left(e^x + e^y\right)$ show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ **(2 marks)**

**c)** (i) In the right angled triangle figure 1, x is increasing at 2cm/s while y is decreasing at 3cm/s. Calculate the rate which z is changing when x = 5 and y = 3cm. **(4 marks)**

(ii) If $z = \tan\left(x^2 - y^2\right)$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ **(2 marks)**

Figure 1

**d)** The total surface areas of a cone of base radius r and perpendicular height h is given by:

$$S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$$

If r and h are each increasing at the rate of 0.25cm/s, find the rate at which S is increasing at the instant when r = 3cm and h = 4cm. **(4 marks)**

**Question Three**

**a)** (i) If $x^2 + y^2 - 2x - 6y + 5 = 0$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3,\ y = 2$ **(4 marks)**

(ii) If $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **(3 marks)**

**b)** (i) A particle moves so that its distance x from a fixed origin at time t is given by $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0$

Solve this equation given that at time t = 0, x = 0 and also $\frac{dx}{dt} = 5$ (noting that $y = Ae^{mt} + Be^{mt}$ is the general solution) **(3 marks)**

(ii) Given the equations a curve as $x = \frac{3t}{1+t},\ y = \frac{t^2}{1+t}$. Find the value of $\frac{dy}{dx}$ when t = 2

**(2 marks)**

c) (i) The radius of curvature R is given by:

$$R = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$$

Find the radius of curvature for the hyperbola xy = 4 at the point x = 2, y = 2. **(3 marks)**

(ii) Given that $\cos^2 y + \sin^2 y = 1$ and that $x = \sin y$ prove that $\frac{d}{dx}\{\sin^{-1} x\} = \frac{1}{\sqrt{1 - x^2}}$ **(2 marks)**

**d)** Find the differential expression with respect to x, $x^5 \sin 2x \cos 4x$ of Differentiate with respect to x $x^5 \sin 2x \cos 4x$

**(3 marks)**

**Question Four**

**a)** (i) Solve the set of equation using the matrix method.

$$2x_1 - 2x + 3x_3 = 2$$
$$x_1 + 3x - x_3 = 11$$
$$2x_1 - 2x_2 + 5x_3 = 3$$

**(4 marks)**

(ii) If $A = (3\times 2)$ matrix $\begin{pmatrix} 5 & 2 \\ 7 & 4 \\ 3 & 1 \end{pmatrix}$ and $B = (2\times 3)$ matrix $\begin{pmatrix} 9 & 2 & 4 \\ -2 & 3 & 6 \end{pmatrix}$ using AB prove that a $a(3\times 2)\times(2\times 3)$ matrix $x = (3\times 3)$ matrix. **(2 marks)**

**b)** (i) If $A = \begin{pmatrix} 5 & 2 & 4 \\ 1 & 3 & 8 \\ 7 & 9 & 6 \end{pmatrix}$ and I is the unit matrix, prove that A.I = A. **(2 marks)**

(ii) If $A = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{pmatrix}$. Find the adjoint of A. **(4 marks)**

**c)** (i) Evaluate $\begin{vmatrix} 3 & 2 & 5 \\ 4 & 6 & 7 \\ 2 & 9 & 2 \end{vmatrix}$ **(2 marks)**

(ii) Solve for 'y' by the use of determinants:

$$x + 2y - 3z = 3$$
$$2x - y - z = 11$$
$$3x + 2y + z = -5$$

**(4 marks)**

**d)** Find the values of K for consistency when

$$x + y - k = 0$$
$$11x - 3y + 11 = 0$$
$$2x + 4y - 8 = 0$$

**(2 marks)**

**Question Five**

**a)** Find the second partial derivatives of the following functions:

$$x^2 + 4x^2y^2 + y^4$$

**(i)** **(3 marks)**

$$\frac{x}{x+y}$$

**(ii)** **(3 marks)**

$$y = \frac{ws^3}{d^4}$$

**b)** (i) , find the percentage increase in y when w increases by 2 per cent, s decreases by 3 per cent and d increases by 1 per cent. **(3 marks)**

$$\frac{dy}{dx} + ay + b = 0$$

(ii) Solve **(2 marks)**

$$y = Ax^2 + Bx$$

**c)** (i) Form the differential equation for **(2 marks)**

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$$

(ii) Solve **(3 marks)**

**d)** In a star connected circuit, current $l_1$, $l_2$, $l_3$ flowing through impendances $Z_1$, $Z_2$, $Z_3$ are given by:

$$i_1 + i_2 + i_3 = 0$$
$$z_1 i_1 - z_2 i_2 = e_1 - e_2$$
$$z_2 i_1 - z_3 i_3 = e_2 - e_3$$

$$z_1 = 10;\ z_2 = 8;\ z_3 = 3;\ e_1 - e_2 = 65;\ e_2 - e_3 = 160;$$

If apply matrix methods to determine the values of $i_1, i_2, i_3$

**(4 marks)**