TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING (BSME Y2 S2)

SMA 2370: CALCULUS IV

END OF SEMESTER EXAMINATION

SERIES: DECEMBER 2014

TIME ALLOWED: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **TWO** printed pages

Question One (Compulsory)

a) State THREE conditions that a function f(x, y) must satisfy for it to be continuous at the point (x_0, y_0). **(3 marks)**

b) If $u = z\sin\frac{y}{x}$, where $x = 3r^2 + 2s, y = 4r - 2s^3,\ z = 2r^2 - 3s^2$ find $\frac{\partial u}{\partial r}$ **(3 marks)**

c) Find the directional derivative of $F = x^2yz^3$ along the curve $e = e^{-u},\ y = \sin u + 1,\ z = u - \cos u$ at the point where u = 0 **(7 marks)**

d) A beam of very low weight, uniform cross-section and length l, simply supported at both ends carries a concentrated load W at the centre. It is known that the deflection at the centre is given by:

$$y = \frac{wl^3}{48EI}$$

where E is Young's modulus and I is a moment of inertia. E is a constant but the following small percentage increases occur 2 $\in$ in W in l and 5 $\in$ in I. Show that the error in y is then negligible. **(5 marks)**

e) Using divergence theorem, evaluate $\iint_s \vec{F}\cdot\hat{n}\,ds$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and s the surface bounded by x = 0, x = 1 y = 0, y = 1 z = 0, z = 1 **(5 marks)**

f) Evaluate $\left(\oint x^2 y\cos x + 2xy\sin x - y^2e^x\right)dx + \left(x^2\sin x - 2ye^x dy\right]\Big)$ along the hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$ **(3 marks)**

g) Prove that $cur\lg rad\phi = \vec{0}$ **(4 marks)**

Question Two

a) Find the maximum and minimum value of $x^2 + y^2 + z^2$ subject to the constraint conditions:

$$\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$$

and z = x + y **(14 marks)**

b) Sketch and name the surface in 3 dimensional space represented by $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ **(6 marks)**

Question Three

a) Find the equations for the:

(i) Tangent line

(ii) Normal plane to the curve $3x^2y + y^2z = -2,\ \ 2xz - x^2y = 3$ at the point (1, -1, 1) **(11 marks)**

b) Expand $x^2y + 3y - 2$ in powers of x – 1 and y + 2 **(9 marks)**
(1 mark)

Question Four

a) Prove that $\int_0^{\pi/2} \ln|\sin x|dx = -\frac{\pi}{2}\ln 2$ **(10 marks)**

b) If $z = z(x, y),\ \ x = r\cos\theta, y = r\sin\theta$, express:

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

in polar coordinates **(10 marks)**

Question Five

a) Verify Stoke's theorem for $\vec{A} = 3y\,\hat{i} - xz\,\hat{j} + yz^2\,\hat{k}$ where S is the surface of the paraboloid $2z = x^2 + y^2$ bounded by z = 2 with the circle C as its boundary **(14 marks)**

b) Find a Fourier series for $f(x) = \cos \alpha x, \quad -\pi \leq x \leq \pi$ where $\alpha \neq 0, \ \pm 1, \ \pm 2, \ \pm 3$ **(6 marks)**