



TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN
ELECTRICAL & ELECTRONIC ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING
BACHELOR OF TECHNOLOGY IN INFORMATION TECHNOLOGY
(BSCE/BSME/BTIT)

SMA 2172/AMA 4101: CALCULUS I

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: OCTOBER 2013
TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Answer Booklet

This paper consist of **FIVE** questions in **TWO** sections **A & B**

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) Given the functions:

$$f(x) = x^3 \quad g(x) = x - 3$$

and

$$(f \circ g)(x)$$

(i) Obtain the composite function

(2 marks)

$$(f \circ g)(x)$$

(ii) Determine the domain and range of (2 marks)

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ c & \text{if } x = 3 \end{cases}$$

b) Determine the value of C so that the function is continuous at $x = 3$ (4 marks)

$$\frac{dy}{dx} = \frac{xy - x\sqrt{1+x^2}}{y\sqrt{1+x^2} - 1 - x^2} \quad y^2 + x^2 = 2y\sqrt{1+x^2}$$

c) Show that (i) (5 marks)

$$\sqrt[5]{33}$$

d) Using the differential estimate correct to 5 decimal places (4 marks)

e) Determine the rate of change of the surface area of a spherical balloon given that its volume is changing at the rate of $2\text{m}^3/\text{s}$ at the instant the radius is 6m (3 marks)

$$y = \log e(x^3 + 2x - 1) \quad \frac{dy}{dx}$$

f) If (4 marks)

SECTION B (Answer any TWO questions from this section)

Question Two

$$f(x) = 5x \quad g(x) = x - 5$$

a) Given that (3 marks)

$$(i) (f \circ g)(x) \quad (ii) (f \circ g)^{-1}(x)$$

b) Evaluate:

$$(i) \lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x}}{x}$$

(4 marks)

$$(ii) \lim_{x \rightarrow 0} \frac{5+x}{x^2+x+1}$$

(3 marks)

$$y = \cos x$$

c) Differentiate from 1st principles (7 marks)

Question Three

$$y = 3x^2 - 4x + 2$$

a) (i) Find the turning points on the graph of (4 marks)

(ii) Sketch the graph of the curve in a(i) (4 marks)

b) A curve is defined parametrically by:

$$y = \frac{2t}{1+t}, \quad x = \frac{1-t^2}{1+t^2}$$

find its gradient at $t = 1$

(6 marks)

$$3y = 6t - 5t^3$$

c) Show that the normal to the curve

drawn at the point K (1, 1/3) passes through the origin

(6 marks)

Question Four

a) Differentiate with respect to x

$$y = \tan^{-1} \frac{2x}{1-x^2}$$

and simplify your answer

(7 marks)

b) The time of swing of a pendulum, T is given by:

$$T = K\sqrt{l}$$

where k is a constant. Determine the percentage change in the time of swing if the length of the pendulum l changes from 32.1cm to 32.0

(5 marks)

c) A rectangular box whose length is one and half its width has a total surface area of 400cm². Find the dimensions of the block that would give it maximum volume

(8 marks)

Question Five

a) The distance x metres travelled by a vehicle in time t seconds after the brakes are applied is given by:

$$x = 20t - \frac{5t^2}{3}$$

$$x = t^3, \quad y = 3t^2 + 8$$

Determine (i) The speed of the vehicle in km/h at the instant the brakes are applied, and(ii) The distance the car travels before it stops

(3 marks)

b) Find the value of C for which the function:

$$f(x) = \begin{cases} x^2 - c^2 & x < 4 \\ Cx + 20 & x \geq 4 \end{cases} \quad (-\infty, \infty)$$

is continuous on

(5 marks)

c) Find the normals to the curve

$$xy + 2x - y = 0$$

that are parallel to the line

$$2x + y = 0$$

(8 marks)