



TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Engineering & Technology

DEPARTMENT OF MATHEMATICS & PHYSICS

INSTITUTIONAL BASED PROGRAMME

UNIVERSITY EXAMINATIONS FOR DEGREE IN:
BACHELOR OF ENGINEERING IN ELECTRICAL & ELECTRONIC/
MECHANICAL/CIVIL ENGINEERING
YR II SEM 1

SMA 2270: CALCULUS III

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: OCTOBER 2013
TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Answer booklet

This paper consists of FIVE questions

Answer question ONE (COMPULSORY) and any other two questions

This paper consist of THREE printed pages

Question One (30 marks)

a) Evaluate the following limits

$$\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$$

(i)

(4 marks)

$$\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$$

(ii)

(4 marks)

b) State the Rolle's Theorem (4 marks)

$$f(x) = x^3 - 6x^2 + 11x - 6$$

c) Verify the validity of Rolle's Theorem for the function (5 marks)

$$y = x^2 \quad y = 2x - x^2$$

d) Find the area of the region enclosed by the parabolas and (6 marks)

$$\frac{df}{dt} \quad t = \frac{\pi}{2} \quad f(x, y) = xy \quad x = \cos t \quad y = \sin t$$

e) Find the value of at if and (7 marks)

Question Two (20 marks)

$$y = \sqrt{r^2 - x^2}$$

a) The semicircle is revolved about the x-axis to generate a sphere. Find the volume of the sphere (6 marks)

$$\frac{dy}{dx} \quad t = -2 \quad f(x, y, z) = \sqrt{x^2 + y^2 + z^2} \quad x = 2 + t, y = -t - 1, z = t$$

b) Find the value of at for the function (5 marks)

c) Evaluate the following integral

(i) $\int_0^2 \int_{x^2}^{2x} (4x + 2) dy dx$ (4 marks)

(ii) $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dx dy dz$ (5 marks)

Question Three (20 marks)

a) State Mean Value Theorem (4 marks)

b) Find the value of c (a, b) guaranteed by Mean Value Theorem for $f(x) = x^2 + 2x - 1, a = 0, b = 1$ (6 marks)

c) Find the Maclaurin series for $f(x) = \ln(x+1)$ up to the term in x^5 (10 marks)

Question Four (20 marks)

$$y = x^2$$

a) The arc of the parabola from (1, 1) to (2, 4) is rotated about the y-axis. Find the area of the resulting surface (10 marks)

$$\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 - 2xy + y^2}{x - y}, x \neq y$$

b) Evaluate (4 marks)

$$f(x) = \cos x \text{ at } a = 2\pi$$

c) Find the Taylor series generated by (6 marks)

Question Five (20 marks)

$$z = x + f(u) \quad u = xy, \quad x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = x$$

a) If $z = x + f(u)$ where $u = xy$, show that (5 marks)

b) Find the volume of the prism whose base is the triangle in the xy -plane bounded by the x -axis and

the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x, y) = 3 - x - y$ (6 marks)

$$f(x, y) = \begin{cases} \frac{xy^2}{x^2 + y^4}, & (x, y) \neq 0 \\ 0, & (x, y) = 0 \end{cases}$$

c) Show that the function $f(x, y)$ is continuous at every point except at the origin. (5 marks)

$$\frac{\partial^3 f}{\partial x \partial y \partial z} \text{ if } f(x, y, z) = xy + yz + zx$$

d) Find (4 marks)