TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING (BSCE/BSME)

SMA 2270/2173: CALCULUS III

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: OCTOBER 2013
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) Verify the mean value Theorem for the function $f(x) = e^x$ in (0, 1) **(5 marks)**

b) Given that $w = x + \frac{y-x}{z-y}$ evaluate the value of $\frac{\delta w}{\delta x} + \frac{\delta w}{\delta y} + \frac{\delta w}{\delta z}$ **(5 marks)**

$$\int_0^2 \int_2^4 \left(x^2 y + xy\right) dy\ dx$$

c) Evaluate the following double integral **(5 marks)**

d) Find a rational number corresponding to the infinite decimal number 1.211111… **(4 marks)**

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

e) show that **(5 marks)**

f) Determine with proof, whether the following sequences converge or diverge

$$a_n = \frac{4}{\sqrt{n+1}}$$

(i) **(3 marks)**

$$a_n = 2 + \cos n\pi$$

(ii) **(3 marks)**

SECTION B (Answer any TWO questions from this section)

Question Two

$\frac{dz}{dt}$ $z = x^2 y + 3xy^4$ $x = \sin 2t$ $y = \cos t$

a) Find , when t =0 given that $_x$ where and **(6 marks)**

b) Show that:

$$\int (\sec x)^h dx = \frac{1}{n-1}\tan x(\sec x)^{n-2} + \frac{n-2}{n-1}\int (\sec x)^{n-2} dx$$

(8 marks)

c) Evaluate the following limits **(3 marks)**

$$\lim_{x\to\infty} \frac{\ln x}{x}$$

(i) **(3 marks)**

$$\lim_{x\to 0} \frac{\ln(\sin x)}{\ln(\tan x)}$$

(ii) **(3 marks)**

Question Three

$$f(x) = \frac{1}{x}$$

a) What are the first five terms of the Taylors series generated by the function **(6 marks)**

$$\sum_{n=1}^{\infty} \frac{x^n}{n}$$

b) Find the interval of convergence of **(6 marks)**

$r = 1 - \cos\theta$

c) Determine the moment of inertia about the y-axis of the region enclosed by the cardial **(8 marks)**

Question Four

$$\lim_{x\to\infty}\left\{\frac{1}{x} - \frac{\log(1+x)}{x^2}\right\}$$

a) (i) Evaluate **(5 marks)**

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos n^{-2} x dx$$

(ii) Show that

b) Find the Maclaurin series for coshx, expressing your result in sigma notation **(5 marks)**

c) State Rolle's Theorem hence find C in the interval (-2, 2) such that f(x) = 0 for the function $f(x) = x^4 - 2x^2$ **(5 marks)**

Question Five

$$f(x) = \frac{1}{x}$$

a) Explain what is meant by continuity of a function hence determine whether the function is continuous within the interval (0, 1) **(6 marks)**

b) Two stationary patrol cars with raclars are 10km apart on a highway. As a truck passes the first patrol car, its peed is clocked at 60km^{-}h^{-1}. Five minutes later when the truck passes the second patrol car. Its speed is clocked at 45kmh^{-1}
Prove that the truck must have exceeded the speed limit of 60kmh-1 at some point during the five minutes. **(6 marks)**

c) Using double integration, determine the volume of solid generated by revolving the ellipse. $\frac{x^2}{d^2} + \frac{y^2}{5^2} = 1$
about x-axis **(8 marks)**