TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY
BACHELOR OF SCIENCE IN CIVIL ENGINEERING
BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING
(BSIT 12J, BSEE 14G, BSME 14G, BSCE 14G)

AMA 4103 SMA 2101: CALCULUS I

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FOUR** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Given the function $f(x) = x^3$ and $g(x) = 3x + 8$

(i) Obtain the composite function $(f \circ g)(x)$ **(2 marks)**
(ii) Determine the domain and range of the function (i) above **(2 marks)**

b) Determine the value of k for the following function to be continuous:

$$f(x) = \begin{cases} 3x^2 + 1 & x \le 1 \\ kx + 1 & x > 1 \end{cases}$$

(2 marks)

c) Find $\frac{dy}{dx}$ if:

(i) $y = \cos(\cos(\cos x^2))$ **(5 marks)**

(ii) $y = x^2 \cot x$ **(2 marks)**

d) Evaluate the following limits

(i) $\lim_{x \to 49} \frac{x - 49}{6 - \sqrt{x - 13}}$ **(5 marks)**

(ii) $\lim_{x \to -2} \frac{x + 2}{x^2 + x - 2}$ **(3 marks)**

e) Differentiate from first principles

$$y = \sqrt{x - 2}$$

(6 marks)

f) The surface area of a spherical cell S, is proportional to the radius r, and is given by:

$S = 4\pi r^2$

Determine the rate of growth of the surface area when r = 10μm, given the radius is growing at 0.1 μm/s **(3 marks)**

Question Two

a) A curve is defined parametrically by:

$$y = \frac{2t}{1+t}, x = \frac{1-t^2}{1+t^2}$$

Find its gradient when t = 1 **(6 marks)**

b) The equation of a curve is given by $y = x^3 - 6x^2 + 9x - 2$

(i) Find the turning points and distinguish between them **(7 marks)**

(ii) Hence sketch the graph of the curve **(3 marks)**

c) Determine the equation of the normal to the curve $3y = 6t - 5t^3$ at (1, 1/3) **(4 marks)**

Question Three

a) A rectangular box whose length is twice its width has total surface area of 300cm^2. Find the dimensions of the box that would give it maximum volume. **(7 marks)**

b) If $y = \tan^{-1}\frac{2x}{1-x^2}$ show that $\frac{dy}{dx} = \frac{2}{1+x^2}$

c) A particle P travels in a straight line AB, its distance x, from A at the end of t seconds being given by:

$$x = 2t^3 - 15t^2 + 36t + 20$$

(i) Find the time(s) at which the particle is stationary and the distance (s) from A when this happens. **(4 marks)**

(ii) Find the time at which the particle attains a constant velocity **(2 marks)**

Question Four

a) Let $f(x) = 2x + 1$ and $g(x) = \frac{x}{3}$ show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ **(7 marks)**

b) Given that $f(0) = 8 \quad f(0) = 5$ and $g(0) = 2 \quad g'(0) = 1$ find the derivative of $h(x)$ at x =0 where:

$$h(x) = \frac{f(x)}{g(x)} + 3x^2 + 4x$$

(4 marks)

c) Determine $\frac{dy}{dx}$ if:

(i) $x^2 + 2xy + y^3 = 5$ **(5 marks)**

(ii) $y = e^{\sqrt[3]{x^2-1}}$ **(4 marks)**

Question Five

a) Determine continuity of a function $f(x)$ at a point x = b **(3 marks)**

b) Define $f(x) = \frac{x^2 + x - 6}{x^2 - 4}$ so that it is continuous at x = 2 **(3 marks)**

c) Find the normal to the curve $xy + 2x - y = 0$ that is parallel to the line 2x + y=0 **(11 marks)**

d) Define the limit of a function f(x) at a point x = a **(3 marks)**