TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN STATISTICS & COMPUTER SCIENCE
BACHELOR OF MATHEMATICS & COMPUTER SCIENCE
(BMCS/BSSC)

AMA 4208: ALGEBRAIC STRUCTURES

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Define the following notions:
- **(i)** Grupoid
- **(ii)** Group
- **(iii)** Semi Group
- **(iv)** Ring
- **(v)** Normal subgroup **(5 marks)**

b) Let G be a cyclic group of order n generated by a, show that G has $\phi(n)$ generators. **(4 marks)**

c) Show that anon empty subset H of G is a subgroup if and only if:

(i) $a, b \in, H \Rightarrow a \cdot b \in H$

(ii) $a \in H \Rightarrow a^{-1} \in H$ **(4 marks)**

d) Show that the algebraic structure (z_5 $+_5$) forms a Abelion group **(4 marks)**

e) Show that the algebraic structure formed by the set of natural numbers under ordinary addition is a semi group **(2 marks)**

f) Given the algebraic structure (z_6, $+_6$), show that $H = \{0,2,4\}$ is its subgroup **(2 marks)**

g) Let R be the set of integers show that the structure of R forms a ring under ordinary addition and multiplication. **(4 marks)**

h) Let R be on a algebraic structure under addition modulo 5 and multiplication modulo 5 respectively. Show that it is indeed a field **(3 marks)**

i) Show that the algebraic structure G{1, -1, i, -1} forms an Abelian group under ordinary multiplication **(2 marks)**

Question Two

a) Show that the set G composed of P_0, P_1, P_2, P_3 of four mappings as of the set of all complex numbers defined by P_0(z) = z, P_1 (z) = -z, P_2 (z) = 1/z and P_3 (2) = -1/z forms an algebraic structure which is an abelion group **(7 marks)**

b) Given the algebraic structure ({1, 2, 3, 4}, X_5). Find its generators **(5 marks)**

c) Let 6:5 T and $T : T \rightarrow \cup$, show that:
   (i) 60T is onto if each 6 and T is onto
   (ii) 60T is one to one if each of 6 and T is one to one **(4 marks)**

d) Given two algebraic structures {1, -1} and {1, -1, i, -1} show that {1, -1} is a subgroup of $\{1, -1, i, -i\}$ (**(4 marks)**

Question Three

a) Let a be an algebraic structure consisting of integers under addition. Let it be even subsets of G. Find the cosets of H and G **(4 marks)**

b) State and proof the Lagrange's theorem **(5 marks)**

c) Given an algebraic structure (H_3, +) where H_3 = {….-3, 0, 3 ….}:
   (i) Find the left and right cosets of H in z where Z is a group under ordinary addition **(4 marks)**
   (ii) Show that the set of cosets forms a group **(4 marks)**

d) Make and complete any cayley table to show a commutative relation under addition modulo 4 of integers **(3 marks)**

Question Four

a) Show that an algebraic structure formed from the set of rational numbers under ordinary addition and multiplication forms a ring with unit element **(7 marks)**

b) Show that $\left(z_{5,} \ +_5, \ x_5\right)$ forms a field **(6 marks)**

c) Show that the polynomial over a ring R forms a ring with respect to addition and multiplication of polynomial **(7 marks)**

Question Five

a) Show that the set of complex numbers C forms a field under the operations of (a + bi) + (c + di) = (a + b) + (c +d)i and (a + bi) . (c + di) = (ac – bd) + (ad + bc) forms a field **(5 marks)**

b) Let $f:\Re \to \Re$ and $g:\Re \to R$ be defined by $f(x) = x + 2$ and $g(x) = \frac{1}{\left(x^2 + 1\right)}$
Find:
(i) $g^o f\ (x)$ **(3 marks)**
(ii) $f^o g\ (x)$ **(3 marks)**
(iii) $(f^o g)^{-1}\ (x)$ **(4 marks)**

c) Show that (G = {0, 1, 2, 3}, $+_4$) is a cyclic group **(3 marks)**

d) Make a cayley table to define an associative binary operation on S = {a, b, c, d}under ordinary addition **(2 marks)**