



TECHNICAL UNIVERSITY OF MOMBASA
Faculty of Applied & Health
Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

**UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN
MECHANICAL ENGINEERING/CIVIL AND BUILDING/ELECTRICAL &
ELECTRONICS ENGINEERING**

SMA 2170: ALGEBRA

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: OCTOBER 2013

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A & B**

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4}n^2(n+1)^2$$

a) Prove by induction that:

(7 marks)

b) Simplify:

$$\frac{\sqrt{1+3x} \quad \sqrt[4]{1+x}}{\left(1+\frac{x}{2}\right)^3}$$

given that powers of x above the first may be neglected (6 marks)

c) Prove that:

$$2 \log_c (a+b) = 2 \log_c a + \log_c \left(1 + \frac{2b}{a} + \frac{b^2}{a^2}\right)$$

(5 marks)

d) The roots of the equation $5x^2 + 7x - 9 = 0$ are α and β . Determine:

$$\frac{1}{\alpha} + \frac{1}{\beta}$$

(i) (2 marks)

$$\alpha^2 + \beta^2$$

(ii) (3 marks)

$$i \left(\frac{1+4i}{1-3i} \right)^2$$

e) Evaluate: (5 marks)

f) The fifth and sixth term of a G.P are 6 and 9 respectively. Determine the Fourth term (2 marks)

SECTION B (Answer any TWO questions from this section)

Question Two

$$\frac{15!}{8! 5!}$$

a) Simplify (3 marks)

$$(x+2y)^6$$

b) Expand by binomial theorem and apply the expansion to evaluate $(1.02)^6$ correct to five places of decimals (7 marks)

c) Express in polar form (5 marks)

$$(5+2j)(4-5j)(2+3j)$$

$$0.\overline{45}$$

d) Express in fractional form and in the lowest form (5 marks)

Question Three

a) Money is invested and accrued interest at a compound interest 6.1% per half year. If shs 300,000 is invested in the account. Find:

- (i) Total value of investment after 5 years (4 marks)
(ii) how many years before the investment exceeds kshs 1,600,000 (5 marks)
- b) The second, fifth and eleventh terms of an arithmetic progression are in geometric progression and the seventh term is 4. Find the first term and common difference (8 marks)
- c) If $a = 3 + i$ and $b = 4 + 2i$, find $3b + 4a - 10i$ (3 marks)

Question Four

- a) Solve correct to 3 decimal places:
 $3x \times 3^{2x-3} = 12$ (5 marks)

- b) Simplify:
 $\sqrt{27} - 2\sqrt{48} + 5\sqrt{75} + \sqrt{147}$ (4 marks)

- (i) ${}^7C_3 \div {}^9C_5$ (3 marks)

- (ii) $(2x + 3yi)(3x + 2yi)$ (3 marks)

- c) If α and β are the roots of $ax^2 + bx + c = 0$, verify that:

- (i) $\alpha + \beta = -b/a$ (3 marks)

- (ii) $\alpha\beta = c/a$ (2 marks)

Question Five

- a) Find the expression $\cos 3\theta$ in terms of $\cos^3 \theta$ and $\cos \theta$ (7 marks)

- b) Expand:
 $\cos^3 \theta$ (7 marks)

- c) Find the remainder when $x^5 - 4x^3 + 2x + 3$ is divided by:
(i) $x - 1$ (2 marks)
(ii) $x + 2$ (2 marks)

$$\frac{\sqrt{2}}{\sqrt{7-\sqrt{2}}} + \frac{12}{\sqrt{7+\sqrt{2}}} = a\sqrt{7} + a\sqrt{2}$$

d) If

(2 marks)