TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN STATISTICS & COMPUTER SCIENCE
BACHELOR OF MATHEMATICS & COMPUTER SCIENCE

AMA 4212: VECTOR ANALYSIS

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FOUR** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) If $\vec{r}_1 = 2i - j + k \quad \vec{r}_2 = i + 3j - 2k \quad \vec{r}_3 = -2i + j - 3k \quad \vec{r}_4 = 3i + 2j + 5k$ Find scalars a, b, c such that $r4 = a\vec{r}_1 + b\vec{r}_2 + c\vec{r}_3$ **(5 marks)**

b) Show that the points, A(-4, 9, 6) B(-1, 6, 6) and C(0, 7, 10) form a right angled isosceles triangle. **(5 marks)**

c) Find the work done in moving an object along vector $\vec{r} = 3i + 2j - 3k$ it the applied force is $\vec{F} = 2i - j - k$

(5 marks)

d) Determine a unit vector perpendicular to the plane of $\vec{A} = 2i - 6j - 3k$ and $\vec{B} = 4i + 3j - k$

(5 marks)

e) A particle moves along a curve whose parametric equations are $x = e^{-t}$ $y = 2\cos 3t$ $z = 2\sin 3t$ where t is the time.

(i) Find velocity and acceleration at time t **(2 marks)**

(ii) Find the magnitude of the velocity and acceleration at t = 0 **(3 marks)**

f) If $\phi(x, y, z) = 3x^2 y - y^3 z^2$ find $\nabla\phi$ at the point (1, -2, -1) **(5 marks)**

Question Two

a) Verify Greens theorem for $\oint_c xydx + (y^2 + 1)dy$ and C is the circle centred origin, radius 2

(10 marks)

b) Evaluate $\int_C \vec{A} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) given that $A = (3x^2 + 6y)i - 14yzj + 20xz^2 k$ along the path x = t y = t^2 z = t^3 **(10 marks)**

Question Three

a) Prove that:

(i) $\nabla(F + G) = \nabla F + \nabla G$ (5 marks)

(ii) $\nabla(FG) = F\nabla G + G\nabla F$ **(5 marks)**

where F and G are differentiable scalars of x, y, z

b) If $A = A_1 i + A_2 j + A_3 K$ $\vec{B} = B_1 i + B_2 j + B_3 k$ $\vec{C} = C_1 i + C_2 j + C_3 j$ show that:

$$\vec{A} \cdot \left(\vec{B} \times \vec{C} \right) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

(5 marks)

c) Prove that $A \cdot \left(\vec{B} \times \vec{C}\right) = B \cdot \left(\vec{C} \times \vec{A}\right) = \vec{C} \cdot \left(\vec{A} \times \vec{B}\right)$ **(5 marks)**

Question Four

A particle moves so that its position vector is given by $\vec{r} = \cos wti + \sin wtj$ where w is a constant show that.

d) Velocity $\vec{v}$ if the particle is perpendicular to $\vec{r}$ **(7 marks)**

e) The acceleration $\vec{\alpha}$ is directed towards the origin and has magnitude proportional to the distance from the origin **(7 marks)**

f) $\vec{r} \times \vec{v} = a$ constant vector **(6 marks)**

Question Five

a) Verify Stokes' theorem given:
$A = (x+y)i + (2y-x)j + z^2k$ and S is the upper surface of the sphere $x^2 + y^2 + z^2 = 1$
(15 marks)

b) Find the projection of $\vec{A} = i - 2j + k$ on the vector $\vec{B} = 4i - 4j + 7k$ **(5 marks)**