TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE MATHEMATICS & COMPUTER SCIENCE (BMCS)

AMA 4319: TEST OF HYPOTHESIS

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: JUNE/JULY 2015
TIME ALLOWED: 2 HOURS

Question One (Compulsory)

a) Define the following terms:
   (i) Type II error **(1 mark)**
   (ii) P value of a test **(1 marks)**

b) Let x be a binomial random variance with parameters (5, θ). Consider testing Ho; $\theta \leq \frac{1}{2}$ vs $H_1; \theta > \frac{1}{2}$. Determine the power function of a test that rejects Ho if and only of all success are observed **(3 marks)**

c) A random sample of 10 high school students who had taken the college board mathematics examination given in the following spring after being in the previous full semester scores for each student shown in the form (fall, spring) are as follows:

(500, 515); (610, 590); (425, 470); (520, 510); (450, 475); (420, 445); (570, 595); (480, 485) (530, 555); (440, 455) Calculate the test station Cs that you would use if you were to assess whether tutoring raises were the mean test score. **(5 marks)**

d) Amcar has the following pairs of figures on number of sales before and after an advertisement (12, 10); (8, 14); (9, 9); (10, 15); (11, 12) calculate the test statistics you would use if you were to test hypothesis on effectiveness of the advertisement **(5 marks)**

e) An Engineer who is studying the tensile strength of a steel alloy intended for use in golf club shafts knows that tensile strength is approximately normally distributed with $\mu = 3500\,psi$ and $\delta = 60\,psi$. A random sample of 12 specimens has a mean tensile strength of $\overline{x} = 3250$. What is the smallest level of significance at which you would be willing to reject the null hypothesis if you were to test the hypothesis that the mean strength is 3500 psi **(4 marks)**

f) A random sample of 1600 workers in region 1 and 1400 workers in region 2 have been obtained to determine whether the population proportions unemployed in the two regions are different. Perform a hypothesis test at the 5% level if the numbers unemployed in the samples were 120 in region 1 and 84 in region 2 **(6 marks)**

Question Two

a) Suppose that $x_1, x_2 ... x_n$ is a random sample from a normal distribution with unknown mean μ and unknown variance σ^2 show that the likelihood ratio test for testing the hypothesis $Ho; \mu = \mu_0$ against $H_1; \mu \neq \mu_0$ is given:

$$L(x) = \left[1 + \frac{n(\overline{xi} - \mu_0)^2}{\sum (xi - \overline{x})^2} \right]^{-n/2}$$

(10 marks)

b) Seniors in the high schools of a city have in the past had a mean score of 490 on a standardized mathematics test. A teacher suggests that seniors will have a higher mean score if they attend tutorials sessions before taking the test. Perform a hypothesis test at the 0.05 level if the scores of a random sample of 35 tutored seniors who take the standardized test have a mean of 510 and standard deviation of 85 **(10 marks)**

Question Three

a) Suppose x_1, x_2, ...x_n is a random sample of size n from a normal distribution with a mean μ and raviance δ^2 where both μ and δ are unknown. We wish to test the hypothesis:

$H_o; \delta^2 = \delta^2 = $ Vs $H_1; \delta^2 \neq \delta^2$ at the α level of significance. Show that the likelihood ratio test is equivalent to the $x^2 \alpha,\ n-1$ test **(10 marks)**

b) The standard deviation of a sample of 15 rivets is 0.008mm. From the past studies it is known that if the standard deviation of the hole diameter exceeds 0.01, there is an unacceptable high probability that the rivet will not fit. From this sample is there strong evidence to indicate that the standard deviation hole diameter exceeds 0.01mm? **(5 marks)**

c) A chain store has been buying hot dogs from a meat-processing company which was sold recently and now is under new management. In the past 90% of all hot dog. Purchased were of high quality. The chain wants to determine whether the percentage is different under the new management. Perform a test at the 1% level of significance if 238 of a random sample of 250 hot dogs are high quality **(5 marks)**

Question Five

a) State and prove Neyman-Pearson Lemma **(10 marks)**

b) A product developer is interested in reducing the drying time of a prime paint. Two formulations of the paint are test: Formulation 9 is the standard chemistry, and the formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation I and another 10 specimens are painted with formulation 2; The 20 specimens are painted in random order. The two sample average drying times are $\overline{x_1} = 121$ minutes and $\overline{x_2} = 112$ minutes respectively. What conclusions can the product developer draw about the effectiveness of the new ingredient using $\alpha = 0.05$? **(10 marks)**

Question Five

a) A study was done to evaluate the relationship between y (the purity of oxygen) in a chemical distillation process and x, the percentage of hydrocarbons that are present in the main condenser of the distillation unit the following values were obtained from the data:

$\hat{B}, = 14.97, \ N = 20, \ S_{xx} = 0.68088, \hat{\delta}^2 = 1.18$ $H_o B1 = 0 \ vs \ H_1; B_1 \neq 0$

Test the hypothesis that

b) The sample correlation coefficient between distance travelled and fuel consumption was conducted. A sample of size 25 was used in the study and the sample correlation. Coefficient was found to be 0.9818. Determine if this sample correlation coefficient is significant at 5% **(7 marks)**

c) Productivity (units produced per day) for a random sample of 10 workers was recorded before and after training. The following data pairs were obtained, the first number in each pair is the before output and the second is the after output. (54, 60); (56, 59); (50; 57) (52, 56); (55, 56); (52, 58); (56, 56); (53, 55); (53, 54); (60, 64) Perform a hypothesis test at the 1 percent level to determine if mean productivity is greater after training than before training **(7 marks)**