THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

UNIVERSITY EXAMINATIONS

DEPARTMENT OF MATHEMATICS AND PHYSICS

SECOND SEMESTER SPECIAL / SUPPLEMENTARY EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING/ MECHANICAL ENGINEERING/BUILDING AND CIVIL ENGINEERING

SMA 2271: ORDINARY DIFFERENTIAL EQUATIONS

DATE: DECEMBER 2011
Time: 2 Hours

INSTRUCTIONS: Answer Question ONE and any other TWO Questions

QUESTION ONE (30 MARKS)

a) State the necessary conditions for a differential equation to be considered linear (3 marks)

Hence with reasons state whether the equation $3\frac{d^3y}{dx^3}+3y\frac{d^2y}{dx^2}+4\frac{dy}{dx}+y=e^{2x}$ is linear (1 mark)

b) Differentiate between general and particular solution of a differential equation. Hence show that the function $f(x)=\left(x^3+c\right)e^{-3x}$ where c is an arbitrary constant is a solution of the differential equation $\frac{dy}{dx}+3y=3x^2e^{-3x}$ (4 marks)

c) Given the differential equation $\left(Ax^2+Bxy+Cy^2\right)dx+\left(Dx^2+Exy+Fy^2\right)dy=0$, show that the equation is exact if $B=2D$ and $E=2C$ (3 marks)

d) Prove that the transformation $u = y^{1-n}$ reduces the equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$ to a linear equation in u and $x.$ Hence solve the equation $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$ (7 marks)

e) Given the differential equation $a_o(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0$ explain each of the following statements in relation to a power series solution.

i) Ordinary point of the equation (1 mark)

ii) Regular singular point of the equation (1 mark)

iii) Hence using Taylor's series expansion, find a power series solution of the equation $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0$ $y(1) = 0, \ \ y^{/}(1) = 4$ (6 marks)

f) Obtain a general solution of the equation $\left(x^2 + 2y^2\right)dx - 2xydy = 0$ (4 marks)

QUESTION TWO (20 MARKS)

a) By separation of variables solve $y\tan x\frac{dy}{dx} = \left(4 + y^2\right)\sec^2 x$. (4 marks)

b) Solve the linear fractional equation $\frac{dy}{dx} = \frac{x + y - 3}{x - y - 1}$ to obtain the general solution. (6 marks)

c) Find the power series solution of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

(10 marks)

QUESTION THREE (20 MARKS)

a) Solve $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$

(7 marks)

b) An object moves with simple harmonic motion on the x axis. Initially it is located at a distance 46 m away from the origin when t=0 and has velocity v=15 m/s and decelerating at $100m/s^2$ directed towards the origin O. find the equation of the position at any time t. (6 marks)

c) Find the particular solution for the initial value problem $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$ if $y(0) = 0$, $y'(0) = -4$.

(7 marks)

QUESTION FOUR (20 MARKS)

a) Solve $\frac{dy}{dx} + y\cot x = \cos x$ to obtain the particular solution given that at $x = \frac{\pi}{2}$

When $y = \frac{5}{2}$.
(5 marks)

b) Obtain a general solution of the equation $\left(x^2 - xy + y^2\right)dx - xydy = 0$.
(7 marks)

c) An electric circuit consists of an inductance of 0.1 henry a resistance of 20 ohms and a condenser of capacitance 25 microfarads. Find the charge q and the current i at any time t, given that the initial conditions are q =0.05 coulombs and $i = \frac{dq}{dt} = 0$ when t =0 if $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E(t)$. (8 marks)

QUESTION FIVE (20 MARKS)

a) Show that $\left(y^3 + 2x\right)dx + \left(3xy^2 + 1\right)dy = 0$ is exact and then find its general solution.

(5 marks)

b) The initial temperature of a body is $53^0 c$ and after 5 minutes its temperature is $45^0 c$, from Newton's law of cooling it is known that the rate of cooling of a body is proportional to the temperature difference between the body and its surrounding room temperature. Use this to predict the temperature of the body after a further 5 minutes given that the room temperature was constant at 21^0C.

(7 marks)

c) Using laplace transform solve $\frac{dx}{dt}+2x=4e^{3t}$ given that at $t=0,\ x=1$.

(8 marks)

THE END